

Operational Risk Assessment of Power Systems with Distributed Energy Resources Using Minimal Cut Sets

by

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Electric power system networks are facing major challenges because of the rapid increase in penetration of distributed energy resources (DERs). Reliability evaluation plays an important role in system analysis, design, upgrades, and operations, especially in bulk power systems. The research presented in this thesis focuses on the evaluation of the composite system reliability under steady state conditions, and also goes a step further towards assessing operational risks in real-time system operations using direct probabilistic analysis techniques. The thesis also examines the reliability and risk improvements that are accrued from penetration of DERs into the power system.

The challenge of using analytical methods in reliability evaluation of composite power systems is the large computational burden involved, to examine all the possible outage events. This thesis presents the mathematical foundations, evaluation procedures, and reliability indices associated with composite power system reliability evaluation using the minimal cut set calculations. The objective of this approach is to evaluate reliability and risk indices for the system and for every load bus in the system. The performance of the system under outage condition of generators, transmission lines, or both, is examined by conducting an appropriate power flow study. An optimal power flow (OPF) is solved to find the system minimal cut sets which are then used to evaluate reliability and operational risk. DER units are incorporated to investigate the enhancement in system reliability and operational risk. The concepts and developed model are illustrated by application to the 24-bus IEEE Reliability Test System.

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Last but not the least, grateful thanks are extended to all the people who stood by and supported me in my academic pursuit.

Dedication

TO MY PARENTS

My ^{الله}LORD! HAVE MERCY UPON THEM AS THEY CHERISHED ME WHEN I WAS LITTLE AND AS THEY
ARE STILL SUPPORTING ME WHEN I AM GROWN UP.

AND

TO MY UNCLE SULEIMAN

WHO RAINS ME WITH LOVE AND KINDNESS

Table of Contents

AUTHOR'S DECLARATION	ii
Abstract	iii
Acknowledgements	iv
Dedication	v
Table of Contents	vi
List of Figures	viii
List of Tables	ix
Nomenclature	x
Chapter 1 Introduction.....	1
1.1 Motivation	1
1.2 Literature Review	2
1.3 Objectives of this Research	5
1.4 Thesis Overview	6
Chapter 2 Background.....	7
2.1 Power System Reliability Basics.....	7
2.1.1 Basic Reliability Concepts.....	9
2.1.2 Fundamental Techniques Used in Reliability Evaluation	11
2.1.3 Method of Minimal Cut Sets	12
2.1.4 Reliability Example	14
2.2 Composite Power System Reliability Evaluation.....	16
2.2.1 Independent Outages	16
2.2.2 Dependent Outages.....	16
2.3 Operational Risk Assessment of Power Systems	17
2.4 Summary	17
Chapter 3 Power System Reliability and Operational Risk Evaluation Using Minimal Cut Sets.....	19
3.1 Introduction	19
3.2 OPF Based Determination of System Minimal Cut Sets.....	20
3.2.1 The OPF Model	23

3.3 Determination of Nodal Minimal Cut Sets.....	24
3.4 IEEE Reliability Test System.....	27
3.5 Reliability Calculations Using Minimal Cut Sets.....	29
3.5.1 Component Unavailability in Steady-State	29
3.5.2 System Unreliability Using Minimal Cut Sets	30
3.5.3 Nodal Unreliability Using Minimal Cut Sets	33
3.6 Operational Risk Calculations Using Minimal Cut Sets	36
3.6.1 Component Unavailability in a Lead Time of 10 hours	36
3.6.2 System Operational Risk Using Minimal Cut Sets.....	39
3.6.3 Nodal Operational Risk Using Minimal Cut Sets.....	41
3.7 Summary	44
Chapter 4 Power System Reliability Analysis with Distributed Energy Resources Penetration.....	46
4.1 Introduction	46
4.2 Mathematical Formulation	46
4.2.1 DER Unavailability	46
4.2.2 OPF Model	47
4.3 Test Results	48
4.3.1 System Unreliability	50
4.3.2 System Operational Risk	53
4.4 Summary	58
Chapter 5 Conclusions and Future Work	59
5.1 Summary of the Thesis.....	59
5.2 Main Contributions.....	60
5.3 Scope for Future Work	61
Appendix A IEEE Reliability Test System Data.....	62
References	65

List of Figures

Figure 2.1: Reliability Aspects.....	7
Figure 2.2: Reliability Assessment Hierarchical Levels.....	8
Figure 2.3: System Operation and Breakdown.....	10
Figure 2.4: Two-state model.....	11
Figure 2.5: Minimal Cut Set Example.....	14
Figure 3.1: Overall Hierarchy to Determine Reliability and Operational Risk	20
Figure 3.2: Schematic for Determining System Minimal Cut Sets	22
Figure 3.3: Schematic for Determining Nodal Minimal Cut Sets	26
Figure 3.4: IEEE Reliability Test System [28].....	28
Figure 3.5: Bus-Wise Unreliability (LOLP) Indices	35
Figure 3.6: Bus-Wise Total Loss of Load	35
Figure 3.7: Generator Unavailability as a Function of the Lead-Time.....	37
Figure 3.8: Transmission Line Unavailability as a Function of the Lead-Time.....	38
Figure 3.9: System Operational Risk as a Function of the Lead-Time	41
Figure 3.10: Nodal Operational Risk as a Function of the Lead-Time (Bus 8).....	43
Figure 3.11: Bus-Wise Operational Risk for Lead-Time = 10 hr.....	44
Figure 3.12: Bus-Wise Total Loss of Load	44
Figure 4.1: IEEE Reliability Test System with DERs.....	49
Figure 4.2: Bus-Wise Unreliability (LOLP) Indices	52
Figure 4.3: Bus-Wise Total Loss of Load	52
Figure 4.4: Generator Unavailability as a Function of the Lead-Time with DERs	54
Figure 4.5: System MCS Unavailability as a Function of the Lead-Time with DERs.....	56
Figure 4.6: System Operational Risk as a Function of the Lead-Time with DERs	56
Figure 4.7: Bus-Wise Operational Risk.....	57
Figure 4.8: Bus-Wise Total Loss of Load	57

List of Tables

Table 1.1: Failure Causes Statistics [1]	1
Table 2.1: Minimal Cut Sets and their Failure Probabilities and System Unreliability of Figure 2.5..	15
Table 3.1 Generators Data and Steady-State Unavailability	29
Table 3.2 Transmission Lines Data and Steady-State Unavailability	30
Table 3.3: Computation of Composite System Reliability using Minimal Cut Sets	32
Table 3.4: Computation of Nodal (Bus-8) Reliability using Minimal Cut Sets	34
Table 3.5: Bus-Wise Unreliability and Loss of Load	35
Table 3.6: Generator Data and Unavailability after 10 Hours	36
Table 3.7: Transmission Lines Data and Unavailability after 10 Hours	38
Table 3.8: Computation of System Operational Risk using Minimal Cut Sets	40
Table 3.9: Computation of Nodal (Bus-8) Operational Risk using Minimal Cut Sets	42
Table 3.10: Bus-Wise Operational Risk for Lead-Time = 10 hr and Total Loss of Load	43
Table 4.1: Data and Steady-State Unavailability for DERs	50
Table 4.2: Computation of Composite System Reliability with DERs using Minimal Cut Sets	51
Table 4.3: Bus-Wise Unreliability and Loss of Load	52
Table 4.4: Generators and DERs Data and Unavailability after 10 Hours	54
Table 4.5: Computation of System Operational Risk with DERs using Minimal Cut Sets	55
Table 4.6: Bus-Wise Operational Risk and Total Loss of Load	57

Nomenclature

Identifiers

i, j	identifier of buses
N	number of buses
NG	number of generator buses
NL	number of load buses
M	order of minimal cut set

Parameters

C_i	minimal cut set i
\bar{C}_i	event that all components of C_i are failed
$G_{i,j}$	conductance of line i, j
P_{fk}	probability of failure for a component k
P_f	probability of system failure
P_f^u	first upper bound of P_f
P_{Loss}	real power loss
\underline{P}_{g_i} and \overline{P}_{g_i}	lower and upper limits of real power generated by a generator i
\underline{P}_{DER_i} and \overline{P}_{DER_i}	lower and upper limits of real power generated by a distributed energy resource i
P_{d_i}	real power demand at bus i
\underline{Q}_{g_i} and \overline{Q}_{g_i}	lower and upper limits of reactive power generated by a generator i
\underline{Q}_{DER_i} and \overline{Q}_{DER_i}	lower and upper limits of reactive power generated by a distributed energy resource i
Q_{d_i}	reactive power demand at bus i
P_{fst}	start up failure probability
$\overline{S}_{i,j}$	maximum capacity of the transmission line between busses i and j
T	lead time (hour)
t_m	instant time
\underline{V}_i and \overline{V}_i	lower and upper limits of voltage magnitude at bus i
Y_{ij}	magnitude of bus admittance element i, j
λ	failure rate

μ	repair rate
δ_i	voltage angle at bus i
θ_i	angle of bus admittance element i, j

Variables

P_{gi}	real power generation at bus i
P_{DER_i}	real power generated by a distributed energy resource at bus i
ΔPD_i	real power load shedding at bus i
Q_{gi}	reactive power generation at bus i
Q_{der_i}	reactive power generated by a distributed energy resource at bus i
ΔQD_i	reactive power load shedding at bus i
$S_{i,j}$	complex power injected in to line between busses i and j
V_i	voltage magnitude at bus i

Chapter 1

Introduction

1.1 Motivation

A power system consists of many components, such as transformers, transmission lines, cables, generators, and loads. The ability of the system to ensure delivery of electrical energy to end users and utility equipment is often subjected to abnormal effects, such as weather conditions, animals, human errors, overload, and ageing that can cause failure of a component. Table 1.1 presents a fourteen-year of historical outage statistics based on an eastern U.S. utility's Outage Management System (OMS) Report in 1996 for some causes that led components to fail [1]. System planners and operators need reliability analyses information on component outages and repair rates in order to ensure system availability and prevent downtimes. Therefore, maintaining continuity and quality of supply, plays an important role in power systems design and operation.

Table 1.1: Failure Causes Statistics [1]

System Outage Causes	Overall System	Transmission system	Distribution System		
			Overall System	Overhead System	Underground System
Animal	40	3	37	35	2
Tree Contact	8260	29	8231	8224	7
Overload	14	2	12	11	1
Work Error	6	0	6	6	0
Equipment Failure	1472	23	1449	1312	137
Lightning	2845	147	2968	2575	123
Accident	140	19	121	89	32
Prearranged	7	1	6	4	2
Customer Problem	122	9	113	109	4
Other	355	10	345	330	15
Total Number of Outages	13261	243	13288	12695	323

One aspect investigated in this thesis is the impact of distributed energy resources (DERs) on power system reliability. DERs are small-scale, modular, energy generation and storage technologies that provide electricity capacity or energy where it is needed (typically producing in the range of 3

kW to 50 MW) [2, 3]. In other words, any technology that is included in distributed generation (DG) and demand-side measures is referred to, as DER [3]. In recent years, DERs have gained attention as a practical option that can significantly improve power quality and reliability without imposing undesirable effects on the environment. Reliability improvements can be made by implementing DERs in power systems which contribute by reducing transmission and distribution congestion, providing spinning reserve, supporting as backup supply balance, and reducing the need for additional system generation, transmission and distribution capacity.

This thesis presents an optimal power flow (OPF) based algorithm to determine the minimal cut sets in order to assess the reliability of a composite power system with DERs. The motivation behind the proposed study is not only to evaluate the long-term reliability, but also to move a step further towards short-term operational risk assessment.

1.2 Literature Review

Up until the 1960's, heuristic methods based on experimentation and rule-of-thumb method were used in determining the reliability of power systems [4]. One of the first recognized works on power system reliability is a study used to assist in cost-reliability tradeoff decisions in the design the power distribution systems [5]. The study considers forced outages of electrical equipment in industrial plants.

In the reliability evaluation of power systems, the states of power system components are usually assumed to be independent and the methods used to calculate the system reliability are based on the multiplication rule of probabilities [6, 7]. The application of probability techniques based on a series of approximate equations to calculate failure rates and component unavailabilities of simple systems is presented in [4, 8]. The mathematical expressions to calculate various measures of reliability are based on series and parallel connections criteria employing probability theory.

Calculations reveal the time of system components that will be subjected to simultaneous occurrences.

One of the most common techniques developed in power system reliability analysis is the Markov Chain or Markov process. However, the large computational burden involved in this method and the rounding errors in the results for large systems, limit the application of the Markov technique [9]. The application of Markov Chains in power system component transitions, from one state to another, for transmission system, is illustrated in [9]. Transmission components are assumed to operate within a 2-state (normal and stormy) weather conditions. The Markov process is used to determine the system failure rate and failure probabilities under the effect of storm associated failures on parallel facilities.

In [10], a Markov cut set method is developed to evaluate reliability of a simple system comprising five components, where the minimal cut sets are determined by using enumeration technique and connectivity analysis. The methodology aims to evaluate the impact of failures from generation and transmission (HL-II) systems on the distribution systems.

A DC-OPF based on Markov cut set method is presented in [7] to determine the impact of adverse weather on the long-term reliability of composite power systems. The proposed method uses DC-OPF approach to determine minimal cut sets which are then used to calculate the reliability indices. Thereafter, the Markov process is applied to the components of the determined minimal cut sets instead of the entire system. The obtained results are compared to the results of Monte Carlo simulation method.

The operation of power systems are subject to uncertainty including random component outages and uncertain load variations. In the past decade, a considerable amount of work has been carried out on assessing power system risk. In [11], a random fuzzy model is presented to evaluate the failure probability of system components due to weather, environment and other operating conditions. A system operational risk assessment method based on credibility theory is developed to

accommodate the two-fold uncertainty combining randomness and fuzziness in power system operations.

In [12], a combined fuzzy and probabilistic method is developed to calculate system risk indices considering system component outage and load uncertainties. The fuzzy membership functions of system component outages are developed using statistical records whereas the system load is modeled using the hybrid method of fuzzy set and Monte Carlo simulation.

In [13], several aspects of operational risk assessment of transmission systems are discussed. The operational risk during different types of adverse weather is estimated once the component failure rates are calculated. The effect of weather parameters on the momentary failure rate and operational risk is discussed. The minimal cut sets are arbitrarily chosen in this paper, without any systematic method.

One aspect of this thesis is to contribute towards enhancing power system reliability associated with DERs. Since, with the rapidly increasing penetration of renewable energy sources and DG technologies, it becomes extremely important to develop reliability techniques that include these alternative sources. The significant reliability benefits associated with renewable and clean DG technologies have made them an attractive option for planners and operators.

In [14], the concepts and techniques for conventional and non-conventional energy sources are introduced. A comprehensive evaluation model of reliability and cost for small isolated systems containing renewable energy sources is presented. Simulation models are used to generate reasonable atmospheric data, evaluate chronological renewable power outputs and combine total energy and load to provide useful system indices.

DG penetration standards in distribution systems are investigated in [15] for eleven utilities and industries to determine the interconnection requirements. The power flow calculations with penetration of DGs in a distribution system are carried out based on the positive sequence model

taking into account different DG characteristic behavior such as voltage profile, harmonics, frequency control, and protection.

A simplified method for reliability evaluation of power system with wind power is presented in [16]. A wind speed model for different locations is developed first, then the method is simplified by determining the minimum multistate representation of a wind farm model in reliability evaluation. The method requires historical wind speed data to be collected over many years, in order to determine the necessary parameters of the wind speed models for the wind farm location.

Further, the value of DGs installed as backup generators can improve the system reliability, depending on the locations of DGs, the number of customers and the sizes of the loads [17]. Enhancement of the reliability is measured by reliability indices that include System Average Interruption Duration Index (SAIDI), Customer Average Interruption Duration Index (CAIDI) and Total Energy Not Supplied (ENS) in [17]. The reliability analysis of a stand-alone photovoltaic system as a function of the hourly solar irradiation in remote areas is presented. The analysis is performed using Monte Carlo simulation methods taking into account the load behavior variability.

In most of the literature reviewed in this section, it is noted that research on power system reliability has not considered the determination of operational risk in the same platform reliability. In the present work, through the use of system and nodal minimal cut sets, these two issues have been considered simultaneously.

1.3 Objectives of this Research

The objectives of this thesis are to:

- Evaluate the reliability of composite power systems in order to help planners to make economic decisions on new investments in generation capacities and transmission lines upgrades.

- Calculate the power system operational risk in real-time in order to help operators maintain the delivery of electricity during system failure and disturbance events such as weather conditions, animals, contingency, load shedding, and human errors.
- Investigate the impact of the DERs on reliability and operational risk and the improvements accrued from their penetrations.

1.4 Thesis Overview

This thesis is organized as follows. Chapter 2 presents a background on power system reliability, and some basic definitions on failure rate, mean time to failure, repair rate, mean time to repair and system availability. It also presents a brief overview on composite system reliability studies and power system operation.

Chapter 3 presents a conceptual framework for determining composite power system reliability and operational risk calculations using the minimal cut sets method. An OPF based model to find the minimal cut sets and the subsequent reliability calculations using minimal cut sets is described in detail. The method is applied to the IEEE Reliability Test System and the reliability and the risk parameters for the system are successfully obtained.

Chapter 4 presents the power system reliability analysis improvements with DERs. The reliability and risk parameters are obtained considering penetration of DERs in the IEEE Reliability Test System, using the proposed mathematical model in Chapter 3.

Finally, Chapter 5 presents the conclusions of the thesis and outlines the future work in this area.

Chapter 2

Background

The electric power system is the most complex system to ever exist. The basic function of a power system is to deliver electricity to customers as reliably and as economically as possible [18]. A quick overview of power system reliability evaluation is presented in this chapter including some basic concepts of power system reliability. Thereafter, a brief review on composite generation and transmission system reliability evaluation is presented. This chapter also presents a brief description of operational risks in power system operation.

2.1 Power System Reliability Basics

Power system reliability is a measure of the ability of the system to meet the load requirements within acceptable standards over a period of several years. In other words, reliability can be defined as the probability that a system/component will perform a required function under stated conditions for a stated period of time [19]. According to North American Electric Reliability Corporation (NERC), reliability is defined as “the degree to which the performance of the elements of the electrical system results in power being delivered to customers within accepted standards and in the amount desired.”

Power system reliability is based on the concepts of system adequacy and system security [20, 21], as shown in Fig. 2.1.

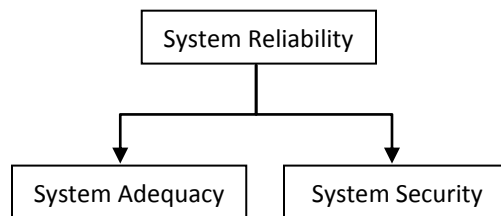


Figure 2.1: Reliability Aspects

Power system adequacy is the ability of the system to supply all energy demand requirements at all times. System adequacy is associated with system steady-state conditions and offers information on future system behavior that can be used in system planning. Security, on the other hand, is the ability of the system to avoid service interruption under sudden disturbances. System security is associated with the dynamic and transient real-time system operations, such as generator and transmission line contingencies and generation uncertainties.

Due to the large-scale and complexity of practical power systems, reliability evaluation can be divided into three zones, i.e., generation, transmission and distribution, organized into three hierarchical levels (HLs) as hierarchical level 1 (HL-I), hierarchical level 2 (HL-II) and hierarchical level 3 (HL-III) as shown in Fig. 2.2 [22]. Reliability studies can be applied to any zone alone, to the combined zones of generation and transmission (HL-II), or to the combined zones of generation, transmission and distribution (HL-III).

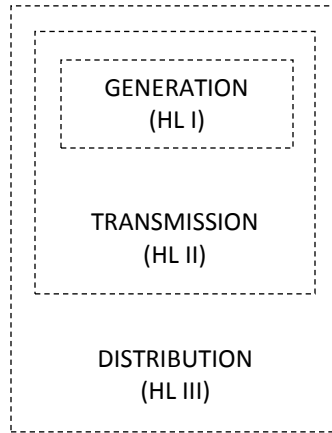


Figure 2.2: Reliability Assessment Hierarchical Levels

Reliability assessment at HL-I considers generating capacity adequacy evaluation to meet the total system load demand. At HL-II, reliability evaluation of the composite system comprising generation and transmission is considered to examine the ability of the system to deliver electrical

energy to all the load points within accepted standards and in the amount desired. Studies also include generation rescheduling and load shedding options. An overall assessment, HL-III, considers all the three zones simultaneously. The reliability assessment at HL-III becomes very difficult because of the large-scale modeling and computation involved. Thus, the distribution system reliability studies are usually performed separately.

2.1.1 Basic Reliability Concepts

A. Failure Rate (λ)

Failure rate is the probability that a component is online during a time interval R . In other words, it is the number of failures of a component per unit measurement of time [23]. Failure rate, as one of the important reliability indices, specifies the rate of system aging. It is generally expressed in failures per hour and often denoted by λ . A failure occurs if any component causes power interruption or abnormal voltage profile.

B. Mean Time to Failure (MTTF)

MTTF is the expected or average time of a component to fail. MTTF is the inverse of the failure rate. From Fig. 2.3, we have,

$$R = MTTF = \frac{1}{\lambda} \quad (2.1)$$

C. Repair Rate (μ)

The repair rate is the probability that a component is recovering and restoring to service again in less than a time R . The repair time represents the time taken to locate the failed component,

diagnose, repair or replace, test, and resume to the system. It is generally expressed in repairs per hour and often denoted by μ .

D. Mean Time to Repair (MTTR)

MTTR is the expected time taken to repair a failed component. MTTR is the inverse of the failure rate. From Fig. 2.3, we have,

$$r = MTTR = \frac{1}{\mu} \quad (2.2)$$

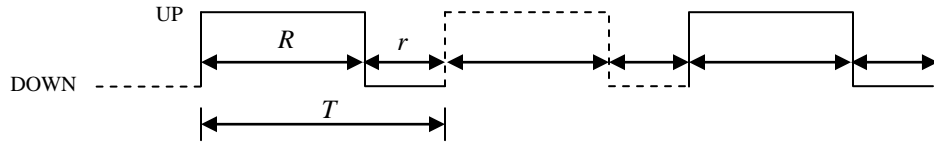


Figure 2.3: System Operation and Breakdown

E. System/Component Availability

Availability is the probability of a system or a component of being in service and being operating. By modeling the system components in series and parallel as interconnections, system availability can be determined.

In order to determine the component availability, let us assume that a component has two states: available and unavailable. The conventional two-state model, shown in Fig. 2.4 is adopted for reliability assessment.

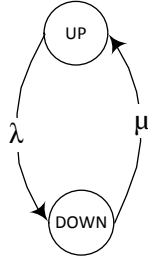


Figure 2.4: Two-state model

Fig. 2.4 indicates the state transition diagram for the two-state device. The model includes an UP (In Service /Available) state and a DOWN (Outage/Unavailable) state. If failures and repairs are exponentially distributed, the probability of a component k on outage at a time $t = T$, given that it was operating successfully at $t = 0$ [18], is

$$P_{f_k} = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)T} \quad (2.3)$$

In steady-state condition, i.e., $t = \infty$, the unavailability or the Forced Outage Rate (FOR) of component k can be obtained [18] as,

$$P_{f_k} = \frac{\lambda}{\lambda + \mu} = FOR \quad (2.4)$$

2.1.2 Fundamental Techniques Used in Reliability Evaluation

Several techniques are developed to improve the reliability of the power system. The criteria applied to assess the composite system reliability can be categorized as deterministic or probabilistic [24].

A. The Deterministic Approach

The deterministic approach is an old and simple method used by system planners to evaluate the system performance and maintain system security in different scenarios based on past experience. The most common deterministic method is the N-1 criterion. Based on this criterion, if a system is able to operate, supply load and remain stable after any single unplanned outage

(one line or one generator) that may occur, the system will be considered reliable. The main advantage of the deterministic approach is its straightforwardness to implement and the easiness to understand. However, the difficulty to determine the degree of system unreliability, which fails under more than one scenario, limits the applications of this method.

B. The Probabilistic Approach

The probabilistic approach provides a better understanding of system behavior and allocation of resources. The benefit of using the probabilistic method is in incorporating uncertain events in the system. The most common types of uncertainties are the components' state, the weather, and the load. Stochastic models are used to represent these uncertainties.

The probabilistic approach is categorized as analytical methods and simulation (Monte Carlo simulation) methods [18]. The analytical methods represent the system behavior by mathematical models and evaluate the system reliability using direct numerical solutions. Some of the analytical methods in use are cut set, Markov, and equivalent method. The simulation methods, on the other hand, estimate the system reliability based on simulating a series of random sampling of scenarios and random behavior.

2.1.3 Method of Minimal Cut Sets

The proposed method in this thesis is based on determining the minimal cut sets for the system generators and lines. As defined in [20], a *cut set* is a set of system components which, when failed, causes failure of the system. System failure refers to the load interruption at any load bus of a composite power system. The minimum subset of any given set of components which causes system failure is known as a *minimal cut set*. The minimal cut set can be defined as a set of system components which, when failed, causes failure of the system but when any one component of the

minimal cut set has not failed, does not cause system failure, i.e., each cut set in the system is in series with other cuts, with parallel components inside a cut. This definition means that all components of a minimal cut set must be in the failure state to cause system failure.

The unreliability or Loss of Load Probability (LOLP) of a power system can be calculated precisely using cut sets, as follows:

- Step-1: Calculate the probability of failure of each component, using (2.3) and (2.4), for operational risk assessment and for determination of FOR, respectively.
- Step-2: Multiply the probability of failure of each individual component that construct a cut set.
- Step-3: Use cut set probabilities, in (2.5) [25], to find the probability of system failure, i.e., LOLP, as follows:

$$P_f = \sum_i P(\bar{C}_i) - \sum_{i < j} P(\bar{C}_i \cap \bar{C}_j) + \sum_{i < j < k} P(\bar{C}_i \cap \bar{C}_j \cap \bar{C}_k) - \dots (-1)^{m-1} \cdot P(\bar{C}_1 \cap \bar{C}_2 \cap \dots \cap \bar{C}_m) \quad (2.5)$$

However, determining cut set probabilities is a difficult and time-consuming exercise for large and complex systems which needs to consider all the cut sets. To overcome the computational complexity, approximations can be made in the evaluation by using the upper bound approximation (first term of Eqn. (2.5)) by summing the minimal cut set probabilities of system failure, as shown in (2.6). The results obtained with this approximation, although not very accurate, allows fast calculation of the LOLP of a system. The degree of inaccuracy introduced, is usually negligible and often within the tolerance associated with the data of the component reliabilities for a system with high values of component reliability.

$$P_f^\mu = \sum_i P(\bar{C}_i) \quad (2.6)$$

2.1.4 Reliability Example

In order to understand the basic concept of the cut set and system LOLP evaluation, Fig. 2.5 describes the logical connections between components. Each block symbolizes a component, and the connections between components describe the state of success or failure of the system. All cut sets that cause a system to fail, should be identified and combined. Any subset of any given set of components which cause a system failure is now called a minimal cut set. In a minimal cut set, all components of the cut set must fail simultaneously in order for the system to fail, and hence the components are in parallel and the minimal cut sets themselves are in series. It is assumed that failure probability of each component is equal to 0.1.

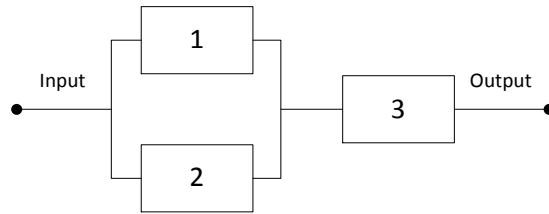


Figure 2.5: Minimal Cut Set Example

From Fig. 2.5, the following minimal cut sets are defined:

C_1 : comprising elements 1 and 2.

C_2 : comprising element 3.

From (2.5) and the reliability of the logic bridge diagram, as shown in Fig. 2.5, the system will fail when either minimal cut set C_1 or C_2 fails, since they are in series. This is mathematically represented as,

$$\begin{aligned} P_f &= P(C_1 \cup C_2) \\ &= P(C_1) + P(C_2) - P(C_1 \cap C_2) \end{aligned}$$

Where,

$$P(C_1) = P_1 \cdot P_2 = 0.1 \times 0.1 = 0.01$$

$$P(C_2) = P_3 = 0.1$$

$$P(C_1 \cap C_2) = P(C_1)P(C_2) = 0.01 \times 0.1 = 0.001$$

therefore,

$$P_f = 0.109$$

The same example is now solved to determine the probability of failure using the minimal cut set method of (2.6). The unreliability or LOLP of the system is now the summation of the minimal cut set unreliabilities:

$$P_f = P(C_1) + P(C_2) = 0.01 + 0.1 = 0.11$$

As seen from this example, the approximate method of minimal cut sets results in an outage probability of 0.11 whereas the exact probability is 0.109. However, the approximation method reduces the computational time and the quantity of analysis that is required, especially for large, complex system.

Table 2.1: Minimal Cut Sets and their Failure Probabilities and System Unreliability of Figure 2.5

Minimal Cut Set	Components in the Cut Set	Minimal Cut Set Failure Probability $P_{fc_i} = \prod_{k=1}^n P_{fk}$
C_1	1,2	0.01
C_2	3	0.1
$P_{f_{system}} = \sum_i P_{fc_i}$		0.11

2.2 Composite Power System Reliability Evaluation

This thesis focuses on HL-II analysis. As mentioned before, HL-II analysis deals with composite power system or bulk power system reliability assessment. Composite power system reliability evaluation examines the ability of the system to deliver electrical energy to the bulk supply points in the amount desired. Components considered in a composite power system reliability study comprise among others, generators, transformers, lines, reactors, relays, and loads. Component outages in a composite system can be characterized as follows [20]:

2.2.1 Independent Outages

Independent outages are those outages of different components which does not affect the probability of outage of the others. Single or multiple independent outages or overlapping outages are the easiest to evaluate and many evaluation methods are available assuming that all the component outages are independent [26]. The research described in this thesis concentrates on the independent outages.

The conventional two-state model, mentioned Section 2.1.1 (Fig. 2.4), represents the probabilistic behavior of most generators and lines, and the basic data required for this model are failure rates and repair rates. For multiple independent outages, the model adopted for reliability assessment can be found by combining the two-state models of each component.

2.2.2 Dependent Outages

Outages are considered dependent when their occurrences are dependent on one or more other outages. Usually, dependent outages are not included in reliability evaluation of composite power systems. Dependent outages can be classified into common mode outages and substation originated outages. A common mode outage occurs when one external cause results in multiple outages, while each individual outage is not affected by the other. An example of the common mode outage is a

single lightning stroke causing tripouts of two or more circuits on a common transmission tower. However, substation originated outage is a forced outage caused by the failure of a component or more inside the substation. An example of substation originated outage is stuck-condition of breakers.

2.3 Operational Risk Assessment of Power Systems

Operational risk assessment is the probability that a system/component will perform a required function under stated conditions during a short period of time. Risk assessment of power systems cover a time scale of hours, which is called the *lead time*, with a known initial operational state [13]. The operating conditions of the system components are uncertain, which render the probability of outage of the components continuously changing. For instance, during severe weather, the failure rate of overhead lines can increase significantly.

Risk and reliability are the two aspects of measuring the ability of the electric power system to meet the load requirements within accepted standards and in the amount desired. Both, risk and reliability are associated with each other. Higher risk means lower reliability, and vice versa.

The first major operational risk assessment was published in 1963 by PJM Interconnection [27]. In this approach, the unit commitment risk is applied to the operational planning of generation units. A procedure is presented for determining operating reserve requirements to maintain a uniform level of risk in the day-to-day operation, taking into account the changing load level, the variability of the load, and the size of units scheduled, so that the spinning reserve capacity in any part of the system can be fully available in any other part of the system [27].

2.4 Summary

In this chapter an attempt is made to present a brief background on power system reliability and its assessment. Reliability definition, concepts of system adequacy and system security, reliability

assessment hierarchical levels, and some basic reliability concepts are briefly discussed. This is followed by distinguishing the difference between the deterministic and the probabilistic approaches in evaluating the system reliability. Thereafter, an example is presented to illustrate the above concepts. Types of component outages in a composite power system are briefly introduced. Finally, a quick overview on operation risk assessment of power systems is presented.

Chapter 3

Power System Reliability and Operational Risk Evaluation Using Minimal Cut Sets

3.1 Introduction

As discussed previously, the main objective of composite generation and transmission system adequacy assessment, HL-II, is to evaluate the ability of a power system to satisfy the load requirements at the major load points. The assessment of adequacy in a composite power system has been considered through various system analysis tools, such as load flow calculations, contingency analysis, generation rescheduling, circuit overload alleviation, load shedding, etc.

Reliability calculations are used to measure the system ability for a long-term performance over several years of a power system, thus covering many operational states. Reliability indices are used by system planners to decide on new investments in generation capacities. On the other hand, risk assessment of power systems covers a time scale of hours, which is called lead-time, with a known initial operational state.

Risk and reliability are the two aspects of measuring the ability of the electric power system to meet the load requirements within accepted standards and in the amount desired. Both, risk and reliability are associated with each other. Higher risk means lower reliability, and vice versa.

The reliability studies presented in this thesis are based on the minimal cut set method. This chapter presents a description of the determination of the system minimal cut sets using optimal power flow (OPF) in Section 3.2 and the determination of the nodal minimal cut sets in Section 3.3. The application of the proposed concepts is made using the IEEE Reliability Test System [28], which is discussed in Section 3.4. In Section 3.5, the computation of system-wide and model reliability indices is presented. Section 3.6 discusses the computation of system-wide and model operational risk assessment and the conclusions of this chapter are summarizing in Section 3.7.

3.2 OPF Based Determination of System Minimal Cut Sets

The system minimal cut sets are used in this chapter to evaluate the reliability and risk indices for the system as a whole, and for every load bus in the system. The performance of the system under the outage condition of generator units, transmission lines, or both, can be examined by conducting an appropriate OPF study. A power flow analysis can be applied to determine the system minimal cut sets but only an OPF is able to determine the nodal minimal cut sets (Section 3.3). Thus, the OPF is used in this thesis to determine both the system and nodal minimal cut sets. The first order outage evaluation checks the outage of one component in the system at a time. The second order outage evaluation examines the simultaneous outage combination of two components. The third and higher order outage evaluations can similarly be defined. The calculation of the indices consists of determining which combinations of component outages result in interruptions and then calculating the probability of these contingencies occurring.

Figure 3.1 presents an overview of the method applied to determine the system-wide and model reliability and operational risk indices.

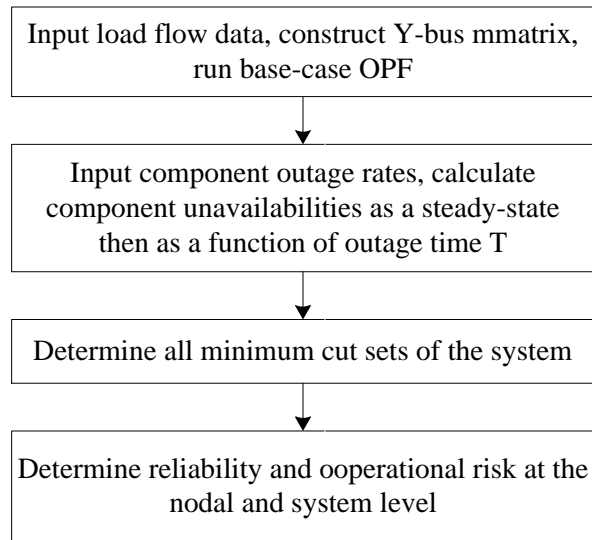


Figure 3.1: Overall Hierarchy to Determine Reliability and Operational Risk

The flow-chart to determine the minimal cut sets of a composite power system up to the preset order is described in Fig 3.2 [7]. The algorithm proceeds as follows:

- Step-1: Select a set of 1st order system components, i.e., each generator or each line is considered individually as a 1st order system component.
- Step-2: Execute the OPF model (discussed in Section 3.2.1) with one component from the above selected set of 1st order system components on outage.
- Step-3: If there is a loss of load at any bus ($\Delta PD_i \neq 0$) then this component is a 1st order minimal cut set. If not, then execute Step-2 with the next component on outage from the selected set, and check for loss of load at any bus. Continue until all components are individually considered to be on outage and hence from the complete list of 1st order minimal cut sets.
- Step-4: Select a set of 2nd order system components, i.e., a combination of two elements, which may be a generator-generator, generator-line, or line-line, pair. Execute Step-2 and Step-3 to determine the complete list of 2nd order minimal cut sets.
- Step-5: Continue Step-1 to Step-4 for higher order of system components. In this work, minimal cut sets up to third order have been considered.

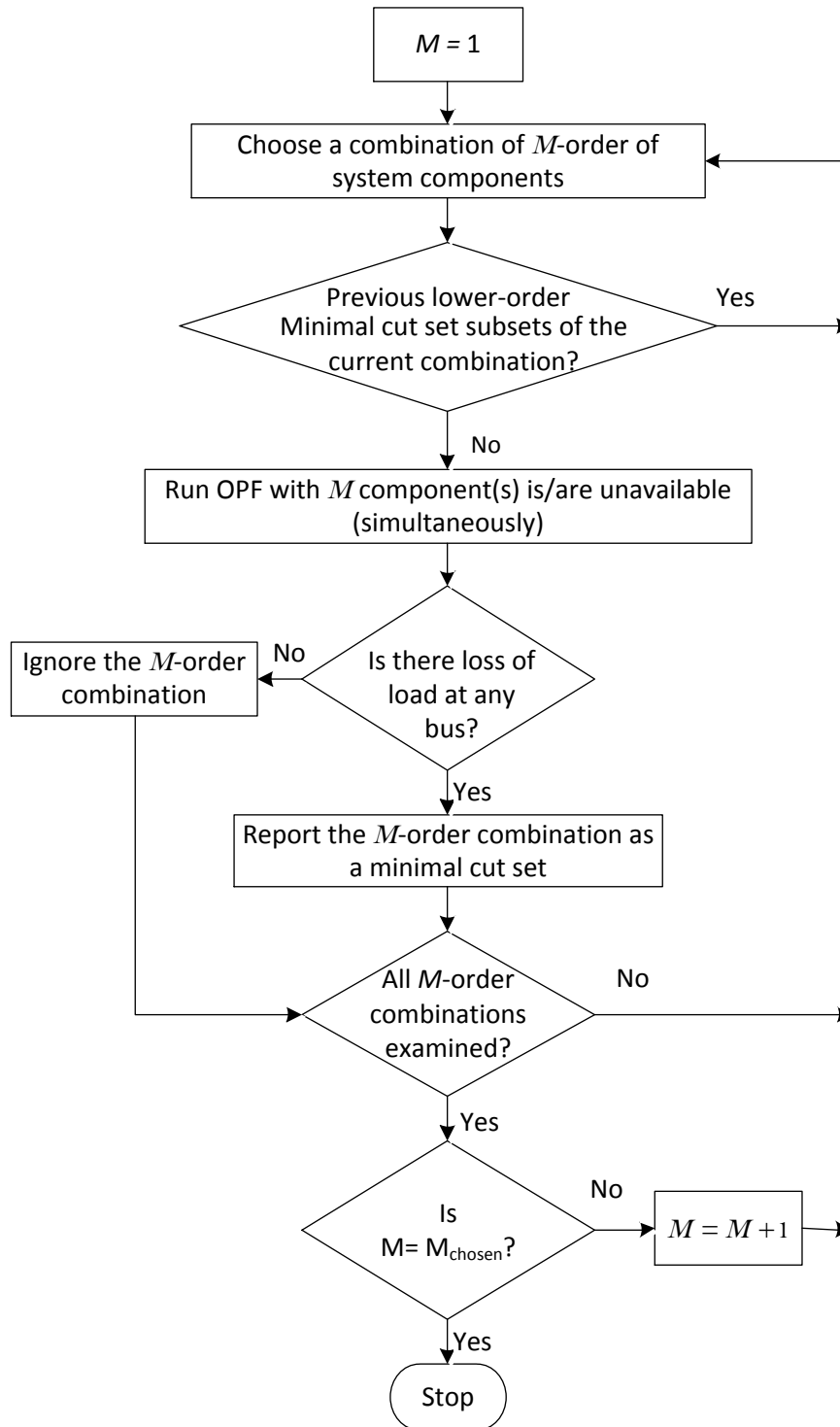


Figure 3.2: Schematic for Determining System Minimal Cut Sets

3.2.1 The OPF Model

The OPF problem is a static, non-linear optimization problem, which determines a set of variables from the network state, load data and system parameters [29]. Optimal values are computed in order to achieve a certain objective such as generation cost or transmission line power loss minimization subject to a number of equality and inequality constraints, such as generation-load balance, generator capacity limits and voltage magnitude limits. In reliability evaluation, the analysis of failure effects should be carried out after the occurrence of a system event.

In a composite power system, after a system event occurs, e.g. the outage of a generator or the tripping of a transmission line, the output of generators is rescheduled first. If the violation of system constraints cannot be alleviated, load shedding is executed.

In order to simulate the OPF model and determine the minimal cut sets, there is a need to select an appropriate objective function for the OPF. In this work, one of the commonly used objectives, the loss minimization objective, is used for the purpose. However, this class of problems is new, and other objective functions can be tested to examine their import on the minimal cut sets, which is left as future work.

The objective function to minimize the active power loss in the transmission system is given below:

$$P_{Loss} = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N G_{i,j} \{V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_j - \delta_i)\} \quad (3.1)$$

The above equation is subjected to the following equality and inequality constraints:

- 1) Nodal active and reactive power balance:

The model active and reactive power balance is ensured by the standard power flow equations (3.2) and (3.3) which are modified to include ΔPD_i and ΔQD_i which are optimization variables. These variables are necessary, as they represent the amount of unsecured active and reactive power at a bus, respectively, that may arise from the outages of various components.

$$Pg_i - Pd_i + \Delta PD_i = \sum_{j=1}^N |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad \forall i \in N \quad (3.2)$$

$$Qg_i - Qd_i + \Delta QD_i = -\sum_{j=1}^N |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \quad \forall i \in N \quad (3.3)$$

2) Limits on bus voltages:

$$\underline{V}_i \leq |V_i| \leq \bar{V}_i \quad \forall i \in N \quad (3.4)$$

3) Limits on active and reactive power generation:

$$\underline{Pg}_i \leq Pg_i \leq \bar{Pg}_i \quad \forall i \in NG \quad (3.5)$$

$$\underline{Qg}_i \leq Qg_i \leq \bar{Qg}_i \quad \forall i \in NG \quad (3.6)$$

4) Capacity limits of transmission line:

$$|S_{i,j}| \leq \bar{S}_{i,j} \quad \forall i, j \in N \quad (3.7)$$

5) Limits on load interruption:

$$\Delta PD_i \leq Pd_i \quad \forall i \in NL \quad (3.8)$$

$$\Delta QD_i \leq Qd_i \quad \forall i \in NL \quad (3.9)$$

3.3 Determination of Nodal Minimal Cut Sets

In order to understand the reliability of serving customers at a specific load bus, it is necessary to determine location specific reliability indices. A method is proposed to obtain the select set of minimal cut sets that result in loss of load at a particular bus. The flow-chart for determining the set of nodal cut sets is shown in Fig. 3.3 and the algorithm is discussed below:

- Step-1: Determine all minimal cut sets of the system, as presented in Section 3.2.

- Step-2: Choose a minimal cut set and execute the OPF (Section 3.2.1) with the associated components of the cut set on outage. If there is a loss of load at bus i ($\Delta PD_i \neq 0$) then this is a minimal cut set of bus i . If there is loss of load at more than one bus, then this cut set is a minimal cut set for all such buses.
- Step-3: Check for all minimal cut sets by repeating Step-2.

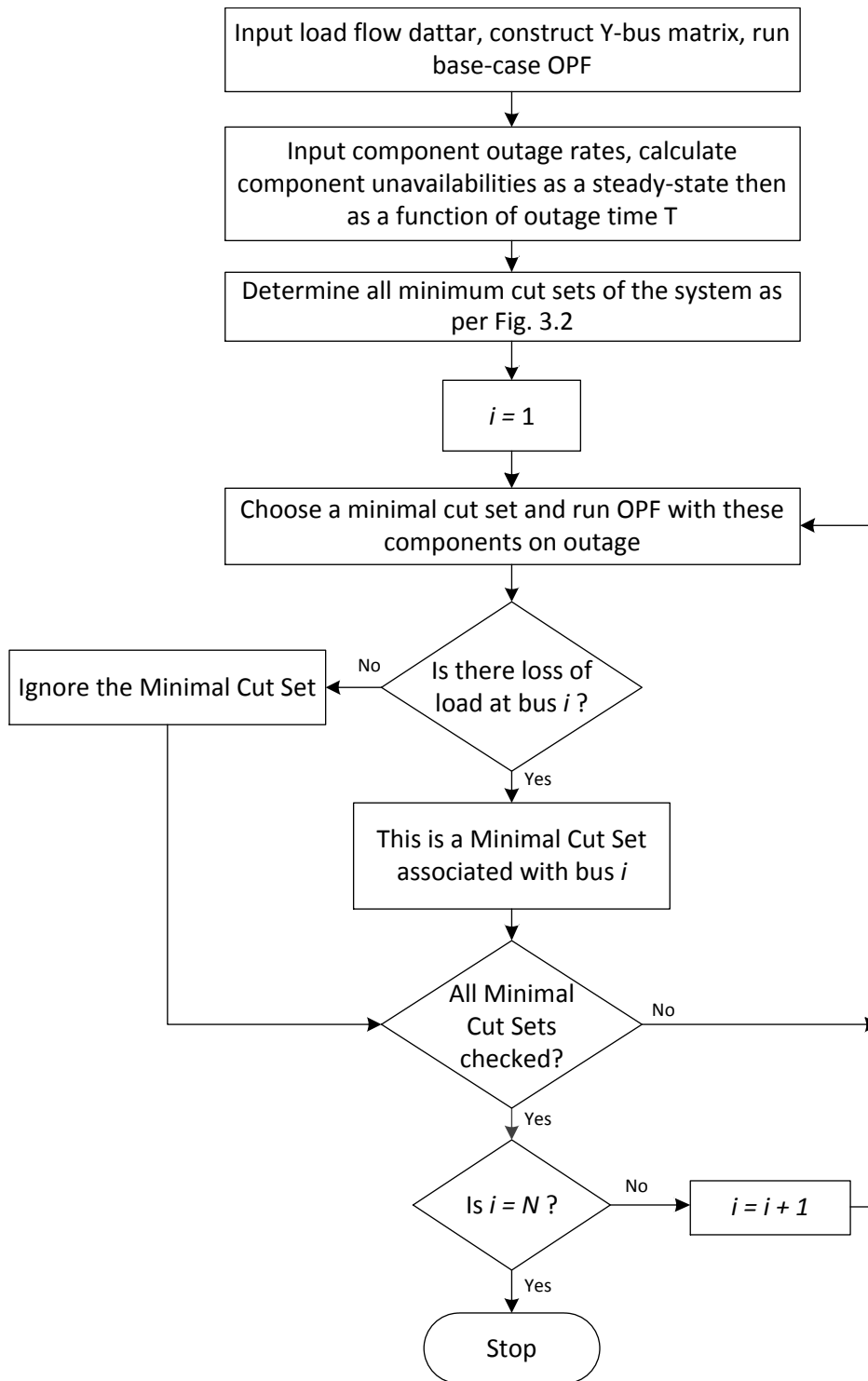


Figure 3.3: Schematic for Determining Nodal Minimal Cut Sets

3.4 IEEE Reliability Test System

In this section, reliability studies are performed on the IEEE Reliability Test System [28] as shown in Fig. 3.4. This system is specially designed by IEEE Task Force on Power System Reliability and it provides all relevant data of lines, generators and outages. The considered test system is programmed and executed in the GAMS environment [30]. There are 32 generators ranging from 12 MW to 400 MW in capacity, 24 buses, and 38 transmission lines and transformers. The system has an annual peak load of 2850 MW and 580 MVar, and the installed generation capacity is 3405 MW. The transmission network consists of 138 kV and 230 kV voltage levels. Relevant data of IEEE Reliability Test System is given in the Appendix.

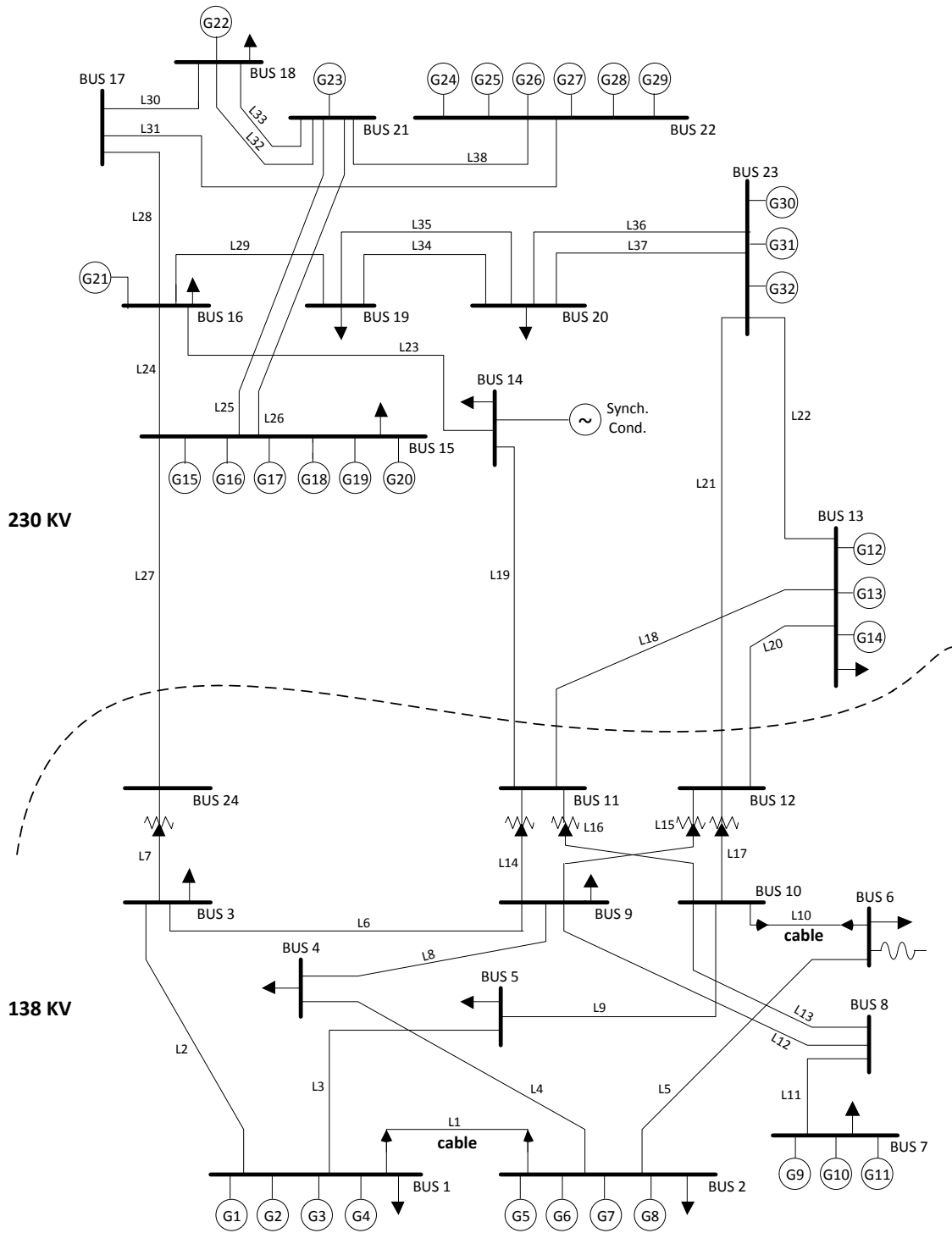


Figure 3.4: IEEE Reliability Test System [28]

3.5 Reliability Calculations Using Minimal Cut Sets

3.5.1 Component Unavailability in Steady-State

Table 3.1 presents the failure rate (λ) and the repair rate (μ) of each generator of the IEEE Reliability Test System, as given in [28]. The resulting steady-state unavailabilities or FOR of each generator are calculated using (2.4) and are given in Table 3.1.

Table 3.1 Generators Data and Steady-State Unavailability

Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	FOR	Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	FOR
1	Gen # 1	2.22E-03	2.00E-02	1.00E-01	17	Gen # 17	3.40E-04	1.67E-02	2.00E-02
2	Gen # 2	2.22E-03	2.00E-02	1.00E-01	18	Gen # 18	3.40E-04	1.67E-02	2.00E-02
3	Gen # 3	5.10E-04	2.50E-02	2.00E-02	19	Gen # 19	3.40E-04	1.67E-02	2.00E-02
4	Gen # 4	5.10E-04	2.50E-02	2.00E-02	20	Gen # 20	1.04E-03	2.50E-02	4.00E-02
5	Gen # 5	2.22E-03	2.00E-02	1.00E-01	21	Gen # 21	1.04E-03	2.50E-02	4.00E-02
6	Gen # 6	2.22E-03	2.00E-02	1.00E-01	22	Gen # 22	9.09E-04	6.67E-03	1.20E-01
7	Gen # 7	5.10E-04	2.50E-02	2.00E-02	23	Gen # 23	9.09E-04	6.67E-03	1.20E-01
8	Gen # 8	5.10E-04	2.50E-02	2.00E-02	24	Gen # 24	5.05E-04	5.00E-02	1.00E-02
9	Gen # 9	8.33E-04	2.00E-02	4.00E-02	25	Gen # 25	5.05E-04	5.00E-02	1.00E-02
10	Gen # 10	8.33E-04	2.00E-02	4.00E-02	26	Gen # 26	5.05E-04	5.00E-02	1.00E-02
11	Gen # 11	8.33E-04	2.00E-02	4.00E-02	27	Gen # 27	5.05E-04	5.00E-02	1.00E-02
12	Gen # 12	1.05E-03	2.00E-02	5.00E-02	28	Gen # 28	5.05E-04	5.00E-02	1.00E-02
13	Gen # 13	1.05E-03	2.00E-02	5.00E-02	29	Gen # 29	5.05E-04	5.00E-02	1.00E-02
14	Gen # 14	1.05E-03	2.00E-02	5.00E-02	30	Gen # 30	1.04E-03	2.50E-02	4.00E-02
15	Gen # 15	3.40E-04	1.67E-02	2.00E-02	31	Gen # 31	1.04E-03	2.50E-02	4.00E-02
16	Gen # 16	3.40E-04	1.67E-02	2.00E-02	32	Gen # 32	8.70E-04	1.00E-02	8.00E-02

Similarly, Table 3.2 presents λ and μ values for each transmission line of the IEEE Reliability Test System [28]. The corresponding FOR of each component, calculated using (2.4), is listed along with.

Table 3.2 Transmission Lines Data and Steady-State Unavailability

Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	FOR	Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	FOR
1	Line # 1	2.74E-05	6.25E-02	4.38E-04	20	Line # 20	4.57E-05	9.09E-02	5.02E-04
2	Line # 2	5.82E-05	1.00E-01	5.82E-04	21	Line # 21	5.94E-05	9.09E-02	6.53E-04
3	Line # 3	3.77E-05	1.00E-01	3.77E-04	22	Line # 22	5.59E-05	9.09E-02	6.15E-04
4	Line # 4	4.45E-05	1.00E-01	4.45E-04	23	Line # 23	4.34E-05	9.09E-02	4.77E-04
5	Line # 5	5.48E-05	1.00E-01	5.48E-04	24	Line # 24	3.77E-05	9.09E-02	4.14E-04
6	Line # 6	4.34E-05	1.00E-01	4.34E-04	25	Line # 25	4.68E-05	9.09E-02	5.15E-04
7	Line # 7	1.26E-06	8.33E-02	1.51E-05	26	Line # 26	4.68E-05	9.09E-02	5.15E-04
8	Line # 8	4.11E-05	1.00E-01	4.11E-04	27	Line # 27	4.68E-05	9.09E-02	5.15E-04
9	Line # 9	3.88E-05	1.00E-01	3.88E-04	28	Line # 28	4.00E-05	9.09E-02	4.39E-04
10	Line # 10	3.77E-05	2.86E-02	1.32E-03	29	Line # 29	3.88E-05	9.09E-02	4.27E-04
11	Line # 11	3.43E-05	1.00E-01	3.42E-04	30	Line # 30	3.65E-05	9.09E-02	4.02E-04
12	Line # 12	5.02E-05	1.00E-01	5.02E-04	31	Line # 31	6.16E-05	9.09E-02	6.78E-04
13	Line # 13	5.02E-05	1.00E-01	5.02E-04	32	Line # 32	4.00E-05	9.09E-02	4.39E-04
14	Line # 14	1.26E-06	8.33E-02	1.51E-05	33	Line # 33	4.00E-05	9.09E-02	4.39E-04
15	Line # 15	1.26E-06	8.33E-02	1.51E-05	34	Line # 34	4.34E-05	9.09E-02	4.77E-04
16	Line # 16	1.26E-06	8.33E-02	1.51E-05	35	Line # 35	4.34E-05	9.09E-02	4.77E-04
17	Line # 17	1.26E-06	8.33E-02	1.51E-05	36	Line # 36	3.88E-05	9.09E-02	4.27E-04
18	Line # 18	4.57E-05	9.09E-02	5.02E-04	37	Line # 37	3.88E-05	9.09E-02	4.27E-04
19	Line # 19	4.45E-05	9.09E-02	4.90E-04	38	Line # 38	5.14E-05	9.09E-02	5.65E-04

3.5.2 System Unreliability Using Minimal Cut Sets

In this subsection, the system minimal cut sets are identified by using the method discussed in Section 3.2 and the results are presented in Table 3.3. The minimal cut sets are determined up to the third-order, in this thesis, in order to keep the computational burden within reasonable limits, and without any loss of generality.

The first-order minimal cut sets are determined considering a single component outage at a time, either a generator or a transmission line. For each outage case, if there is loss of load at a bus, the particular component on outage, becomes a first-order minimal cut set. As observed from Table 3.3, there is no first-order minimal cut set in the IEEE Reliability Test System.

The second-order minimal cut sets are determined considering the simultaneous outage of two components of the system, i.e. two generators, two transmission lines, or one generator and one transmission line. For each outage case, if there is loss of load at a bus, the two components on simultaneous outage form a second-order minimal cut set, if neither of them is a first-order minimal cut set.

In Table 3.3, it is seen that there are 20 second-order minimal cut sets where both components are generators, 8 minimal cut sets where both components are transmission lines, and 3 second-order minimal cut sets of generator-line pairing. The corresponding unavailability, grouped by component and order of minimal cut sets is also presented in Table 3.3.

In the same way, the third-order minimal cut sets for the IEEE Reliability Test System are also determined and the corresponding unavailability, grouped by component and order is presented in Table 3.3.

Once the unavailabilities are obtained for each minimal cut set group, the system unreliability can be determined using (2.6). In the IEEE Reliability Test System under study, the system unreliability or LOLP is found to be 0.15621662.

Table 3.3: Computation of Composite System Reliability using Minimal Cut Sets

Component	Order of Cut Set	Set of Components in Minimal Cut Set	Minimal Cut Set Unavailability	Unavailability by Component and order of Minimal Cut Set	
Generators Only	1st	None	0	0	
	2nd	(12,22),(12,23),(12,32),(13,22),(13,23),(13,32),(14,22),(14,23),(14,32),(20,22),(20,23),(21,22),(21,23),(22,23),(22,30),(22,31),(22,32),(23,30),(23,31),(23,32)	(6.000E-3),(6.000E-3),(4.000E-3),(6.000E-3),(6.000E-3),(4.000E-3),(6.000E-3),(6.000E-3),(4.000E-3),(4.800E-3),(4.800E-3),(4.800E-3),(4.800E-3),(4.800E-3),(1.440E-2),(4.800E-3),(4.800E-3),(9.600E-3),(4.800E-3),(4.800E-3),(9.600E-3)	0.12	
	3rd	(1,9,22),(1,9,23),(1,10,22),(1,10,23),(1,11,22),(1,11,23),(1,20,32),(1,21,32),(1,30,32),(1,31,32),(2,9,22),(2,9,23),(2,10,22),(2,10,23),(2,11,22),(2,11,23),(2,20,32),(2,21,32),(2,30,32),(2,31,32),.....,(29,30,32),(29,31,32),(30,31,32)	(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4),(3.200E-4),(3.200E-4),(3.200E-4),(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4),(3.200E-4),(3.200E-4),(3.200E-4),(3.200E-4),(4.800E-5),.....,(3.200E-5),(3.200E-5),(3.200E-5),(1.280E-4)	0.035981	
Transmission Lines Only	1st	None	0	0	
	2nd	(2,7),(2,27),(3,9),(4,8),(5,10),(11,13),(12,13),(19,23)	(8.768E-9),(2.994E-7),(1.461E-7),(1.828E-7),(7.211E-7),(1.719E-7),(2.520E-7),(2.335E-7)	EPS	
	3rd	(1,4,10),(1,6,7),(1,6,27),(1,8,10),(1,10,11),(1,10,14),(1,10,15),(3,4,5),(4,5,7),(4,5,9),(4,5,13),(4,5,14),(4,5,15),(4,5,16),(4,5,17),(4,5,18),(4,5,20),(4,5,21),(4,5,22),(4,5,25),(4,5,26),.....,(25,26,28),(29,34,35),(29,36,37)	(2.57E-10),(2.86E-12),(9.78E-11),(2.37E-10),(1.98E-10),(8.69E-12),(8.69E-2),(9.18E-11),(3.67E-12),(9.46E-11),(1.22E-10),(3.67E-12),(3.67E-12),(3.67E-12),(1.22E-10),(1.22E-10),(1.59E-10),(1.50E-10),(1.25E-10),.....,(9.36E-11),(1.16E-10),(9.71E-11),(7.77E-11)	EPS	
Generators & Transmission Lines	1st	N/A	0	0	
	2nd	(22,11),(23,11),(32,11)	(4.108E-5),(4.108E-5),(2.739E-5)	0.00010955	
	3rd	1 Gen + 2 Lines	(1,6,7),(1,6,27),(2,6,7),(2,6,27),(3,6,7),(3,6,27),(4,6,7),(4,6,27),(5,1,10),(5,6,7),(5,6,27),(6,1,10),(6,6,7),(6,6,27),(7,1,10),(7,6,7),(7,6,27),(8,1,10),(8,6,7),(8,6,27),.....,(32,23,29),(32,24,28),(32,31,38)	(6.53E-10),(2.231E-8),(6.53E-10),(2.231E-8),(1.31E-10),(4.462E-9),(1.31E-0),(4.462E-9),(5.770E-8),(6.53E-10),(2.231E-8),(5.770E-8),(6.53E-10),(2.231E-8),(1.154E-8),(1.31E-10),(4.462E-9),(1.154E-8),(1.31E-10),(4.462E-9),.....,(1.628E-8),(1.456E-8),(3.061E-8)	EPS
		2 Gens + 1 Line	(9,10,11),(9,10,13),(9,11,11),(9,11,13),(9,22,7),(9,22,17),(9,22,27),(9,23,7),(9,23,27),(10,11,11),(10,11,13),(10,22,7),(10,22,17),(10,22,27),(10,23,7),(10,23,27),(11,22,7),(11,22,17),(11,22,27),(11,23,7),.....,(31,32,30),(31,32,31),(31,32,38)	(5.478E-7),(8.032E-7),(5.478E-7),(8.032E-7),(7.233E-8),(7.233E-8),(2.470E-6),(7.233E-8),(2.470E-6),(5.478E-7),(8.032E-7),(7.233E-8),(2.470E-6),(7.233E-8),(2.470E-6),(7.233E-8),(2.470E-6),(7.233E-8),(7.233E-8),(2.470E-6),(7.233E-8),(2.470E-6),(7.233E-8),.....,(1.285E-6),(2.168E-6),(1.807E-6)	0.000123312
SYSTEM UNRELIABILITY (LOLP)				0.15621662	

*EPS: Very small number

3.5.3 Nodal Unreliability Using Minimal Cut Sets

As discussed in Section 3.3, the algorithm can be applied to find the nodal unreliability or model LOLP using minimal cut sets. For illustration, in Table 3.4 the set of components in minimal cut sets, the minimal cut set unavailabilities, and the corresponding unavailability grouped by component and order of minimal cut sets are specifically identified for Bus-8 of the test system. The minimal cut sets of Bus-8 are the subsets of the system minimal cut sets. As seen in Table 3.4, there is one minimal cut set of the second-order of generators (23, 32) and it is a subset of the system minimal cut sets. The nodal unreliability or LOLP of Bus-8 is determined to be 0.034262393 in the same way that the system unreliability or LOLP is obtained in Section 3.5.2 using (2.6).

In the IEEE Reliability Test System under study, the bus-wise unreliabilities (LOLPs) and the loss of load at all load buses in the system are calculated as shown in Table 3.5, Fig. 3.5 and Fig. 3.6. It is noted that Bus-8 has the highest unserved load in the system but the nodal unreliability index (model LOLP) at this bus is comparatively low and hence the risk of loss of load is low. On the other hand, Bus-3 has a low value of unserved load but a higher unreliability index (higher LOLP), indicating high risk.

Table 3.4: Computation of Nodal (Bus-8) Reliability using Minimal Cut Sets

Component	Order of Cut-Set	Set of Components in Minimal Cut Set	Minimal Cut Set Unavailability	Unavailability by Component and order of Minimal Cut Set
Generators Only	1st	None	0	0
	2nd	(23,32)	9.60E-03	9.60E-03
	3rd	(1,9,22),(1,9,23),(1,10,22),(1,10,23), (1,11,22),(1,11,23),(2,9,22),(2,9,23), (2,10,22),(2,10,23),(2,11,22),(2,11,23), (3,9,22),(3,9,23),(3,9,32),(3,10,22), (3,10,23),(3,10,32),(3,11,22),(3,11,23),,(29,30,32),(29,31,32),(30,31,32)	(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4), (4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4), (4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4), (9.600E-5),(9.600E-5),(6.400E-5),(9.600E-5), (9.600E-5),(6.400E-5),(9.600E-5),(9.600E-5),,(3.200E-5),(3.200E-5),(1.280E-4)	2.45E-02
Transmission Lines Only	1st	None	0	0
	2nd	(11,13), (12,13)	(1.719E-7),(2.520E-7)	EPS
	3rd	(11,23,29)	6.97E-11	EPS
Generators & Transmission Lines	1st	N/A	0	0
	2nd	(22,11),(23,11),(32,11)	(4.108E-5),(4.108E-5),(2.739E-5)	1.10E-04
	2nd	None	0	0
	3rd	(12,13,11),(12,14,11),(12,20,11),(12,21,11), (12,30,11),(12,31,11),(13,14,11),(13,20,11), (13,21,11),(13,30,11),(13,31,11),(14,20,11), (14,21,11),(14,30,11),(14,31,11),(20,32,29), (20,32,31),(20,32,38),(21,32,25),(21,32,26)	(8.559E-7),(8.559E-7),(6.847E-7),(6.847E-7), (6.847E-7),(6.847E-7),(8.559E-7),(6.847E-7), (6.847E-7),(6.847E-7),(6.847E-7),(6.847E-7), (6.847E-7),(6.847E-7),(6.847E-7),(1.366E-6), (2.168E-6),(1.807E-6),(1.647E-6),(1.647E-6)	EPS
NODAL UNRELIABILITY (NODAL LOLP)				0.034262393

*EPS: Very small number

Table 3.5: Bus-Wise Unreliability and Loss of Load

Bus No.	Total Loss of Load (p.u.)	Unreliability (LOLP)	Bus No.	Total Loss of Load (p.u.)	Unreliability (LOLP)
3	28.086	0.087241337	13	0.369	0.000125
4	43.99	0.094343378	14	20.182	0.066774809
5	3.17	0.035982497	15	0.27	0.0144
6	18.496	0.06815277	18	4.483	0.014403272
7	3.805	0.001608419	19	4.198	0.009603269
8	53.388	0.034262393	20	2.949	0.000128006
9	1.853	0.000128001			

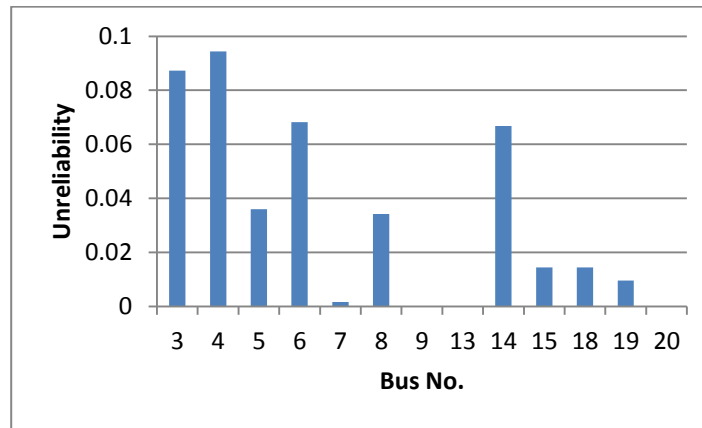


Figure 3.5: Bus-Wise Unreliability (LOLP) Indices

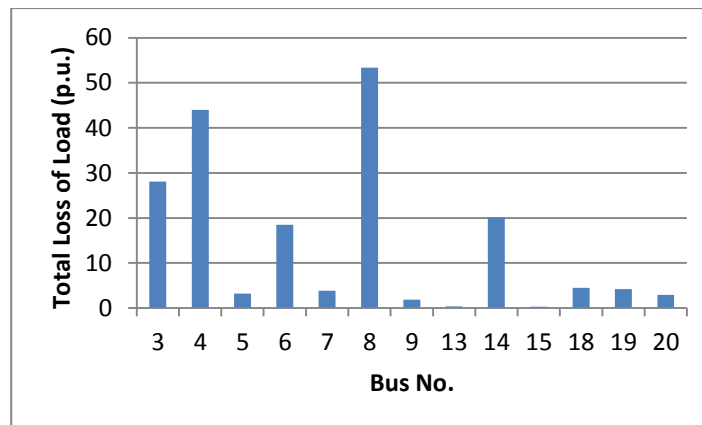


Figure 3.6: Bus-Wise Total Loss of Load

3.6 Operational Risk Calculations Using Minimal Cut Sets

3.6.1 Component Unavailability in a Lead Time of 10 hours

In Table 3.6, λ and μ of each generator of the IEEE Reliability Test System [28], and the resulting component unavailabilities for a lead-time of 10 hours, calculated using (2.3), are presented. The lead-time of 10 hours intends to examine the probability of a component to be unavailable at the end of the hour-10, assuming that the component is available at the start of that lead-time.

Table 3.6: Generator Data and Unavailability after 10 Hours

Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	Unavailability After 10 hrs	Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	Unavailability After 10 hrs
1	GEN#1	2.22E-03	2.00E-02	1.99E-02	17	GEN#17	3.40E-04	1.67E-02	3.13E-03
2	GEN#2	2.22E-03	2.00E-02	1.99E-02	18	GEN#18	3.40E-04	1.67E-02	3.13E-03
3	GEN#3	5.10E-04	2.50E-02	4.50E-03	19	GEN#19	3.40E-04	1.67E-02	3.13E-03
4	GEN#4	5.10E-04	2.50E-02	4.50E-03	20	GEN#20	1.04E-03	2.50E-02	9.17E-03
5	GEN#5	2.22E-03	2.00E-02	1.99E-02	21	GEN#21	1.04E-03	2.50E-02	9.17E-03
6	GEN#6	2.22E-03	2.00E-02	1.99E-02	22	GEN#22	9.09E-04	6.67E-03	8.76E-03
7	GEN#7	5.10E-04	2.50E-02	4.50E-03	23	GEN#23	9.09E-04	6.67E-03	8.76E-03
8	GEN#8	5.10E-04	2.50E-02	4.50E-03	24	GEN#24	5.05E-04	5.00E-02	3.97E-03
9	GEN#9	8.33E-04	2.00E-02	7.52E-03	25	GEN#25	5.05E-04	5.00E-02	3.97E-03
10	GEN#10	8.33E-04	2.00E-02	7.52E-03	26	GEN#26	5.05E-04	5.00E-02	3.97E-03
11	GEN#11	8.33E-04	2.00E-02	7.52E-03	27	GEN#27	5.05E-04	5.00E-02	3.97E-03
12	GEN#12	1.05E-03	2.00E-02	9.49E-03	28	GEN#28	5.05E-04	5.00E-02	3.97E-03
13	GEN#13	1.05E-03	2.00E-02	9.49E-03	29	GEN#29	5.05E-04	5.00E-02	3.97E-03
14	GEN#14	1.05E-03	2.00E-02	9.49E-03	30	GEN#30	1.04E-03	2.50E-02	9.17E-03
15	GEN#15	3.40E-04	1.67E-02	3.13E-03	31	GEN#31	1.04E-03	2.50E-02	9.17E-03
16	GEN#16	3.40E-04	1.67E-02	3.13E-03	32	GEN#32	8.70E-04	1.00E-02	8.24E-03

Figure 3.7 shows the of generator unreliabilities as a function of the lead-time. It is assured that the unavailabilities are linearly varying with the lead-times.

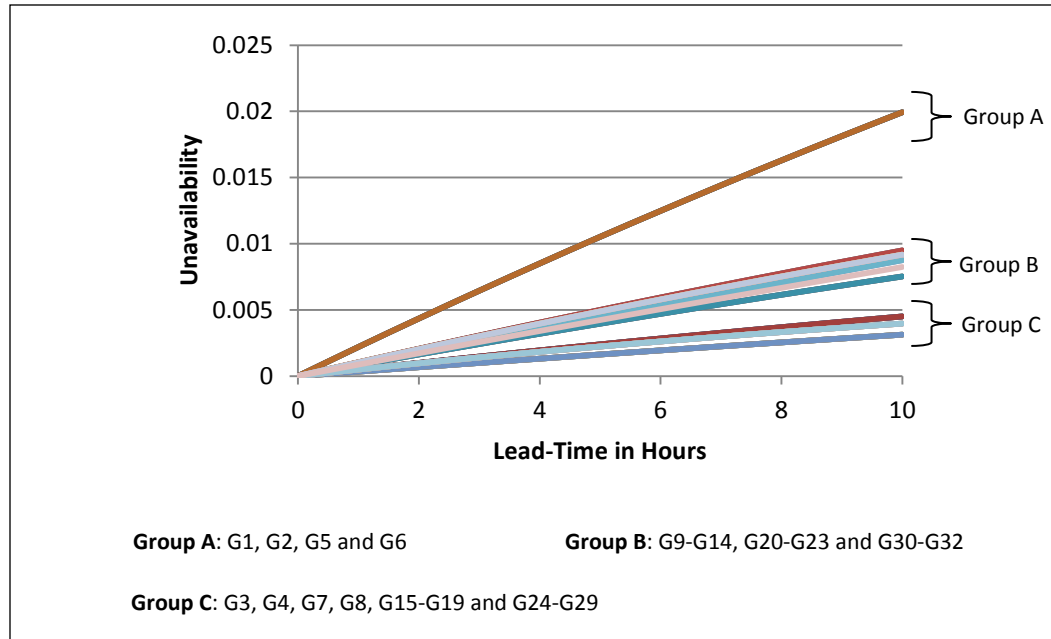


Figure 3.7: Generator Unavailability as a Function of the Lead-Time

Similarly, Table 3.7 presents λ and μ values for each transmission line of the IEEE Reliability Test System [28], and the corresponding unavailabilities for a lead-time of 10 hours calculated using (2.3).

Table 3.7: Transmission Lines Data and Unavailability after 10 Hours

Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	Unavailability After 10 hrs	Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	Unavailability After 10 hrs
1	LINE#1	2.74E-05	6.25E-02	2.04E-04	20	LINE#20	4.57E-05	9.09E-02	3.00E-04
2	LINE#2	5.82E-05	1.00E-01	3.68E-04	21	LINE#21	5.94E-05	9.09E-02	3.90E-04
3	LINE#3	3.77E-05	1.00E-01	2.38E-04	22	LINE#22	5.59E-05	9.09E-02	3.67E-04
4	LINE#4	4.45E-05	1.00E-01	2.81E-04	23	LINE#23	4.34E-05	9.09E-02	2.85E-04
5	LINE#5	5.48E-05	1.00E-01	3.46E-04	24	LINE#24	3.77E-05	9.09E-02	2.47E-04
6	LINE#6	4.34E-05	1.00E-01	2.74E-04	25	LINE#25	4.68E-05	9.09E-02	3.07E-04
7	LINE#7	1.26E-06	8.33E-02	8.52E-06	26	LINE#26	4.68E-05	9.09E-02	3.07E-04
8	LINE#8	4.11E-05	1.00E-01	2.60E-04	27	LINE#27	4.68E-05	9.09E-02	3.07E-04
9	LINE#9	3.88E-05	1.00E-01	2.45E-04	28	LINE#28	4.00E-05	9.09E-02	2.62E-04
10	LINE#10	3.77E-05	2.86E-02	3.28E-04	29	LINE#29	3.88E-05	9.09E-02	2.55E-04
11	LINE#11	3.43E-05	1.00E-01	2.16E-04	30	LINE#30	3.65E-05	9.09E-02	2.40E-04
12	LINE#12	5.02E-05	1.00E-01	3.17E-04	31	LINE#31	6.16E-05	9.09E-02	4.05E-04
13	LINE#13	5.02E-05	1.00E-01	3.17E-04	32	LINE#32	4.00E-05	9.09E-02	2.62E-04
14	LINE#14	1.26E-06	8.33E-02	8.52E-06	33	LINE#33	4.00E-05	9.09E-02	2.62E-04
15	LINE#15	1.26E-06	8.33E-02	8.52E-06	34	LINE#34	4.34E-05	9.09E-02	2.85E-04
16	LINE#16	1.26E-06	8.33E-02	8.52E-06	35	LINE#35	4.34E-05	9.09E-02	2.85E-04
17	LINE#17	1.26E-06	8.33E-02	8.52E-06	36	LINE#36	3.88E-05	9.09E-02	2.55E-04
18	LINE#18	4.57E-05	9.09E-02	3.00E-04	37	LINE#37	3.88E-05	9.09E-02	2.55E-04
19	LINE#19	4.45E-05	9.09E-02	2.92E-04	38	LINE#38	5.14E-05	9.09E-02	3.37E-04

It is seen from Fig. 3.8 that the transmission line unreliabilities increase with lead-times, though they are no longer linearly dependent, unlike the generators.

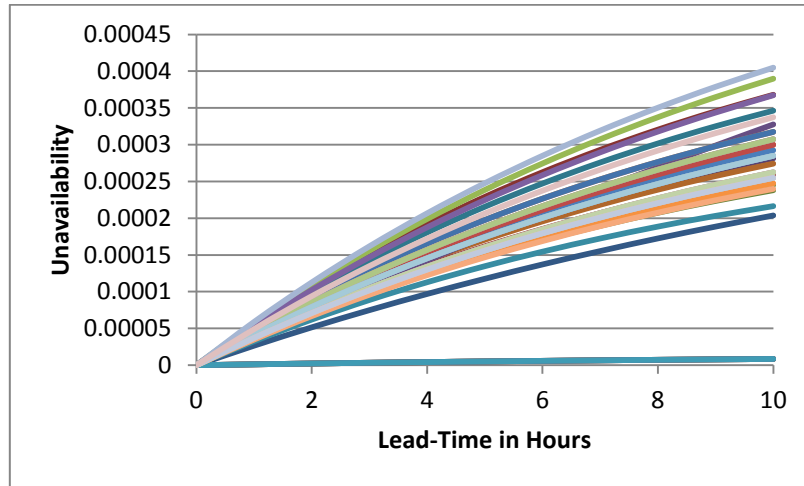


Figure 3.8: Transmission Line Unavailability as a Function of the Lead-Time

3.6.2 System Operational Risk Using Minimal Cut Sets

In this subsection, the system minimal cut sets and their unavailabilities are used to determine the system operational risk using the proposed method. The results obtained are presented in Table 3.8. The operational risk is determined using equation (2.3), for a lead-time of 10 hours after the event, assuming that the system loads remain the same during this period. The same set of minimal cut sets, obtained in Table 3.3 are used for this purpose.

Figure 3.9 presents the plots of system operational risk as a function of the lead-time. It is observed that the operational risk is rather low up to 4 hours of lead-time but thereafter increases exponentially. This implies that in order to prevent the outages to turn into major catastrophe, there need to be restored within typically 4 hours. The system operational risk is found to be 0.001755074 for the lead-time of 10 hours.

Table 3.8: Computation of System Operational Risk using Minimal Cut Sets

Component	Order of Cut Set	Set of Components in Minimal Cut Set	Minimal Cut Set Unavailability at $t=10$ hr	Unavailability by Component and order of Minimal Cut Set
Generators Only	1st	None	0	0
	2nd	(12,22),(12,23),(12,32),(13,22),(13,23),(13,32),(14,22),(14,23),(14,32),(20,22),(20,23),(21,22),(21,23),(22,23),(22,30),(22,31),(22,32),(23,30),(23,31),(23,32)	(8.31E-05),(8.31E-05),(7.82E-05),(8.31E-05),(8.31E-05),(7.82E-05),(8.31E-05),(8.31E-05),(8.03E-05),(8.03E-05),(8.03E-05),(8.03E-05),(7.67E-05),(8.03E-05),(8.03E-05),(7.21E-05),(8.03E-05),(8.03E-05),(7.21E-05)	0.00159648
	3rd	(1,9,22),(1,9,23),(1,10,22),(1,10,23),(1,11,22),(1,11,23),(1,20,32),(1,21,32),(1,30,32),(1,31,32),(2,9,22),(2,9,23),(2,10,22),(2,10,23),(2,11,22),(2,11,23),(2,20,32),(2,21,32),(2,30,32),(2,31,32),.....,(29,30,32),(29,31,32),(30,31,32)	(1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6),(1.506E-6),(1.506E-6),(1.506E-6),(1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6),(1.506E-6),(1.506E-6),(1.506E-6),(1.506E-6),(1.506E-6),.....,(2.996E-7),(2.996E-7),(6.930E-7)	0.000150549
Transmission Lines Only	1st	None	0	0
	2nd	(2,7),(2,27),(3,9),(4,8),(5,10),(11,13),(12,13),(19,23)	(3.14E-09),(1.13E-07),(5.84E-08),(7.31E-08),(1.13E-07),(6.87E-08),(1.01E-07),(8.33E-08)	EPS
	3rd	(1,4,10),(1,6,7),(1,6,27),(1,8,10),(1,10,11),(1,10,14),(1,10,15),(3,4,5),(4,5,7),(4,5,9),(4,5,13),(4,5,14),(4,5,15),(4,5,16),(4,5,17),(4,5,18),(4,5,20),(4,5,21),(4,5,22),(4,5,25),(4,5,26),.....,(25,26,28),(29,34,35),(29,36,37)	(1.88E-11),(4.76E-13),(1.72E-11),(1.73E-11),(1.44E-11),(5.69E-13),(5.69E-13),(2.32E-11),(8.30E-13),(2.39E-11),(3.09E-11),(8.30E-13),(8.30E-13),(8.30E-13),(2.92E-11),(2.92E-11),(3.80E-11),(3.58E-11),(2.99E-11),.....,(2.48E-11),(2.07E-11),(1.66E-11)	EPS
Generators & Transmission Lines	1st	N/A	0	0
	2nd	<div><div>1 Gen + 1 Line</div><div>(22,11),(23,11),(32,11)</div></div>	(1.90E-06),(1.90E-06),(1.78E-06)	EPS
	3rd	<div><div>1 Gen + 2 Lines</div><div>(1,6,7),(1,6,27),(2,6,7),(2,6,27),(3,6,7),(3,6,27),(4,6,7),(4,6,27),(5,1,10),(5,6,7),(5,6,27),(6,1,10),(6,6,7),(6,6,27),(7,1,10),(7,6,7),(7,6,27),(8,1,10),(8,6,7),(8,6,27),.....,(32,23,29),(32,24,28),(32,31,38)</div></div>	(4.65E-11),(1.679E-9),(4.65E-11),(1.679E-9),(1.05E-11),(3.79E-10),(1.05E-11),(3.79E-10),(1.330E-9),(4.65E-11),(1.679E-9),(1.330E-9),(4.65E-11),(1.679E-9),(3.01E-10),(1.05E-11),(3.79E-10),(3.01E-10),(1.05E-11),(3.79E-10),.....,(5.98E-10),(5.35E-10),(1.125E-9)	EPS
		<div><div>2 Gens + 1 Line</div><div>(9,10,11),(9,10,13),(9,11,11),(9,11,13),(9,22,7),(9,22,17),(9,22,27),(9,23,7),(9,23,27),(10,11,11),(10,11,13),(10,22,7),(10,22,17),(10,22,27),(10,23,7),(10,23,27),(11,22,7),(11,22,17),(11,22,27),(11,23,7),.....,(31,32,30),(31,32,31),(31,32,38)</div></div>	(1.225E-8),(1.796E-8),(1.225E-8),(1.796E-8),(5.61E-10),(5.61E-10),(2.024E-8),(5.61E-10),(2.024E-8),(1.225E-8),(1.796E-8),(5.61E-10),(5.61E-10),(2.024E-8),(5.61E-10),(2.024E-8),(5.61E-10),(2.024E-8),(5.61E-10),.....,(1.813E-8),(3.059E-8),(2.549E-8)	EPS
SYSTEM OPERATIONAL RISK				0.001755074

*EPS: Very small number

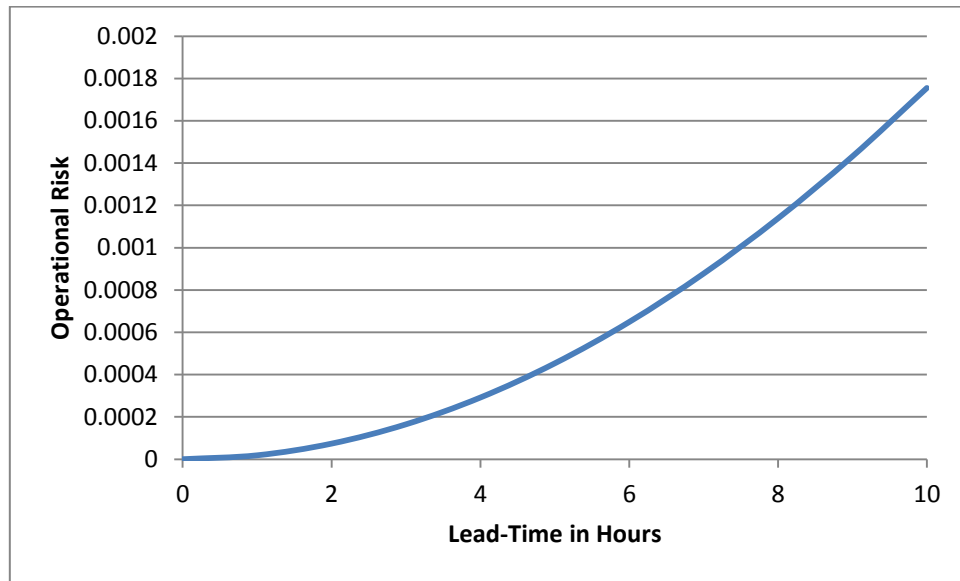


Figure 3.9: System Operational Risk as a Function of the Lead-Time Neglecting Changes in System Operational Conditions

3.6.3 Nodal Operational Risk Using Minimal Cut Sets

The minimal cut sets and their unavailabilities are used to determine the nodal operational risk for bus-8 of the test system, which are listed in Table 3.9. The calculations have also been carried out to find all the unavailabilities of the minimal cut sets and the nodal operational risk of all load buses that suffer from loss of load.

Table 3.9: Computation of Nodal (Bus-8) Operational Risk using Minimal Cut Sets

Component	Order of Cut-Set	Set of Components in Minimal Cut Set	Minimal Cut Set Unavailability at $t=10$ hr	Unavailability by Component and order of Minimal Cut Set
Generators Only	1st	None	0	0
	2nd	(23,32)	7.21E-05	7.21E-05
	3rd	(1,9,22),(1,9,23),(1,10,22),(1,10,23), (1,11,22),(1,11,23),(2,9,22),(2,9,23), (2,10,22),(2,10,23),(2,11,22),(2,11,23), (3,9,22),(3,9,23),(3,9,32),(3,10,22), (3,10,23),(3,10,32),(3,11,22),(3,11,23),,(29,30,32),(29,31,32),(30,31,32)	(1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6), (1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6), (1.312E-6),(1.312E-6),(1.312E-6),(1.312E-6), (2.966E-7),(2.966E-7),(2.791E-7),(2.966E-7), (2.966E-7),(2.966E-7),(2.791E-7),(2.966E-7), (2.966E-7),(2.966E-7),(2.966E-7),(2.966E-7),,(2.996E-7),(2.996E-7),(6.930E-7)	8.43265E-05
Transmission Lines Only	1st	None	0	0
	2nd	(11,13), (12,13)	(6.87E-08),(1.01E-07)	EPS
	3rd	(11,23,29)	1.57E-11	EPS
Generators & Transmission Lines	1st	N/A	0	0
	2nd <small>1 Gen + 1 Line</small>	(22,11),(23,11),(32,11)	(1.90E-06),(1.90E-06),(1.78E-06)	5.57E-06
	3rd <small>1 Gen + 2 Lines</small>	None	0	0
	3rd <small>2 Gens + 1 Line</small>	(12,13,11),(12,14,11),(12,20,11),(12,21,11), (12,30,11),(12,31,11),(13,14,11),(13,20,11), (13,21,11),(13,30,11),(13,31,11),(14,20,11), (14,21,11),(14,30,11),(14,31,11),(20,32,29), (20,32,31),(20,32,38),(21,32,25),(21,32,26)	(1.950E-8),(1.950E-8),(1.884E-8),(1.884E-8), (1.884E-8),(1.884E-8),(1.884E-8),(1.950E-8),(1.884E-8), (1.884E-8),(1.884E-8),(1.884E-8),(1.884E-8), (1.884E-8),(1.884E-8),(1.884E-8),(1.926E-8), (3.059E-8),(2.549E-8),(2.323E-8),(2.323E-8)	EPS
NODAL OPERATIONAL RISK				0.000162615

*EPS: Very small number

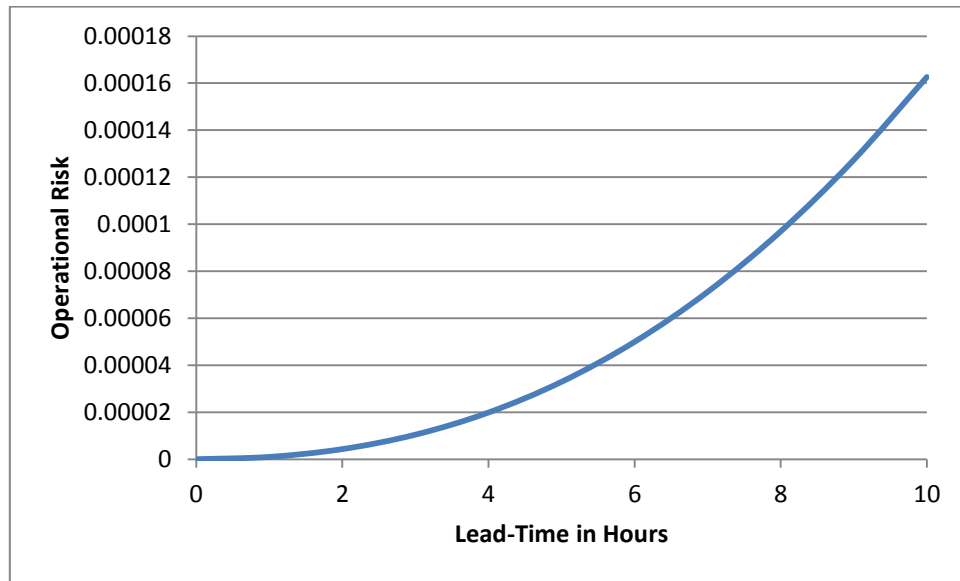


Figure 3.10: Nodal Operational Risk as a Function of the Lead-Time (Bus 8) Neglecting Changes in System Operational Conditions

Table 3.10: Bus-Wise Operational Risk for Lead-Time = 10 hr and Total Loss of Load

Bus No.	Total Loss of Load (p.u.)	Operational Risk	Bus No.	Total Loss of Load (p.u.)	Operational Risk
3	28.086	0.001144302	13	0.369	8.552E-07
4	43.99	0.001222769	14	20.182	0.000715437
5	3.17	7.66341E-06	15	0.27	0.00007665
6	18.496	0.000864114	18	4.483	7.67208E-05
7	3.805	4.88885E-06	19	4.198	7.21861E-05
8	53.388	0.000162615	20	2.949	6.93448E-07
9	1.853	6.93083E-07			

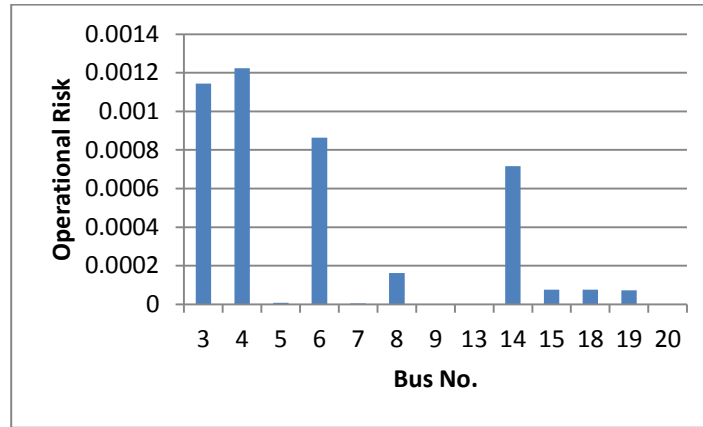


Figure 3.11: Bus-Wise Operational Risk for Lead-Time = 10 hr

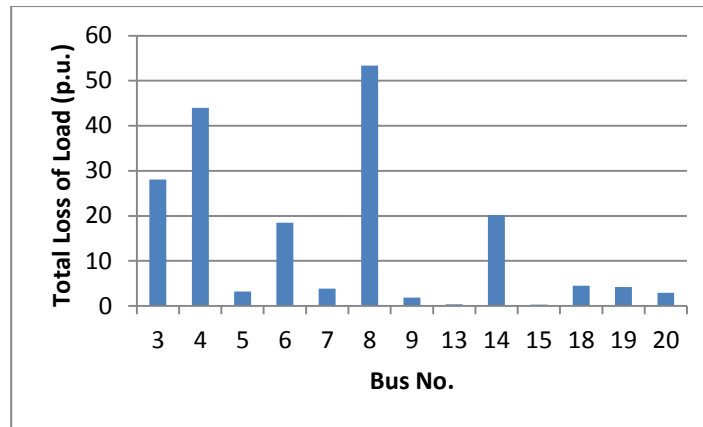


Figure 3.12: Bus-Wise Total Loss of Load

3.7 Summary

In this chapter, the concept of minimal cut sets is applied to evaluate the composite system and nodal unreliabilities. The proposed method is applied to the IEEE Reliability Test System by conducting an appropriate OPF to obtain the system-wide minimal cut sets. Once the component unavailabilities are calculated using failure and repair rates data, the minimal cut set probabilities are calculated which are then implemented in the probability equation to evaluate system-wide and nodal unreliabilities indices.

Whereas this chapter uses a comprehensive reliability and operational risk assessment in composite power systems to predict the performance of the power system over several years (planning) and during a specific time (operation), the next chapter introduces the impact of DERs on reliability and operational risk of the composite power system based on same approach discussed in this chapter.

Chapter 4

Power System Reliability Analysis with Distributed Energy Resources Penetration

4.1 Introduction

One objective of this thesis is to evaluate the improvement in power system reliability with DERs while satisfying network constraints. As described in Chapters 2, the power system is liable to system failures and disturbances, such as load shedding, contingency, weather conditions, animals and human errors. Therefore, maintaining reliability to deliver electricity to all points of utilization within acceptable standards is an important issue for systems design, planning, and operation.

This chapter presents an analysis to study the reliability improvements from the penetration of DERs in composite electrical power systems. In Section 4.2, the mathematical framework for determining the minimal cut sets and subsequently the system reliability and operational risk, with penetration of DER is described. Reliability and risk calculations and results for the system and load buses are obtained by using the proposed method, by application to the IEEE Reliability Test System, and are presented in Section 4.3. Finally, in Section 4.4 a summary is presented.

4.2 Mathematical Formulation

In this section, the mathematical model for system reliability and operational risk due to the addition of DERs is presented.

4.2.1 DER Unavailability

As described in Chapter 2 (Section 2.1.1), the probability of failure of a DER can be obtained as follows:

1. Steady-state probability of failure:

$$P_{f_{DER}} = \frac{\lambda_{DER}}{\lambda_{DER} + \mu_{DER}} = FOR_{DER} \quad (4.1)$$

2. Time-dependent probability of failure:

$$P_{f_{DER}} = P_{fst} + \left(\frac{\lambda_{DER}}{\lambda_{DER} + \mu_{DER}} - \frac{\lambda_{DER}}{\lambda_{DER} + \mu_{DER}} e^{-(\lambda_{DER} + \mu_{DER})(T - t_m)} \right) \quad (4.2)$$

In (4.2), it is assumed that the DER is only available when it is needed, which means, the DER serves as a supplemental reserve with a response time of t_m hours. This is a generic representation, and for $t_m = 0$, the DER can be considered to be in continuous service.

4.2.2 OPF Model

The objective function to minimize the active power loss in the transmission system is considered, as given before:

$$P_{Loss} = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N G_{i,j} \{V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_j - \delta_i)\} \quad (4.3)$$

Some of the equality and inequality constraints are as follow:

1) Nodal active and reactive power balance:

The active and reactive power balance equations are modified by including P_{DER_i} and Q_{DER_i} .

These variables represent the active and reactive power generated by a DER at a bus, respectively.

$$Pg_i + P_{DER_i} - Pd_i + \Delta PD_i = \sum_{j=1}^N |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \quad \forall i \in N \quad (4.4)$$

$$Qg_i + Q_{DER_i} - Qd_i + \Delta QD_i = - \sum_{j=1}^N |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \quad \forall i \in N \quad (4.5)$$

2) Active and reactive power generation limits of the DERs:

$$\underline{P}_{DER_i} \leq P_{DER_i} \leq \overline{P}_{DER_i} \quad \forall i \in NL \quad (4.6)$$

$$\underline{Q}_{DER_i} \leq Q_{DER_i} \leq \overline{Q}_{DER_i} \quad \forall i \in NL \quad (4.7)$$

The remaining constraints, discussed in Chapter 3 (Section 3.2.1), are also included.

4.3 Test Results

The IEEE Reliability Test System presented in Chapter 3 is used here from the reliability analysis study. In this test system, it is assumed that DER1 and DER2 are located at bus 3 and bus 4, respectively (Fig. 4.1). The locations of the DER placements are selected based on load buses with lowest reliabilities as obtained in Chapter 3 that result. Further, the number of DERs and their sizes are arbitrarily chosen. The rated power of DER1 and DER2 are assumed to be 25 MW and 15 MW, respectively. It is further assumed that the failure and repair rate of the two DERs are 0.001 per hour and 0.02 per hour, respectively.

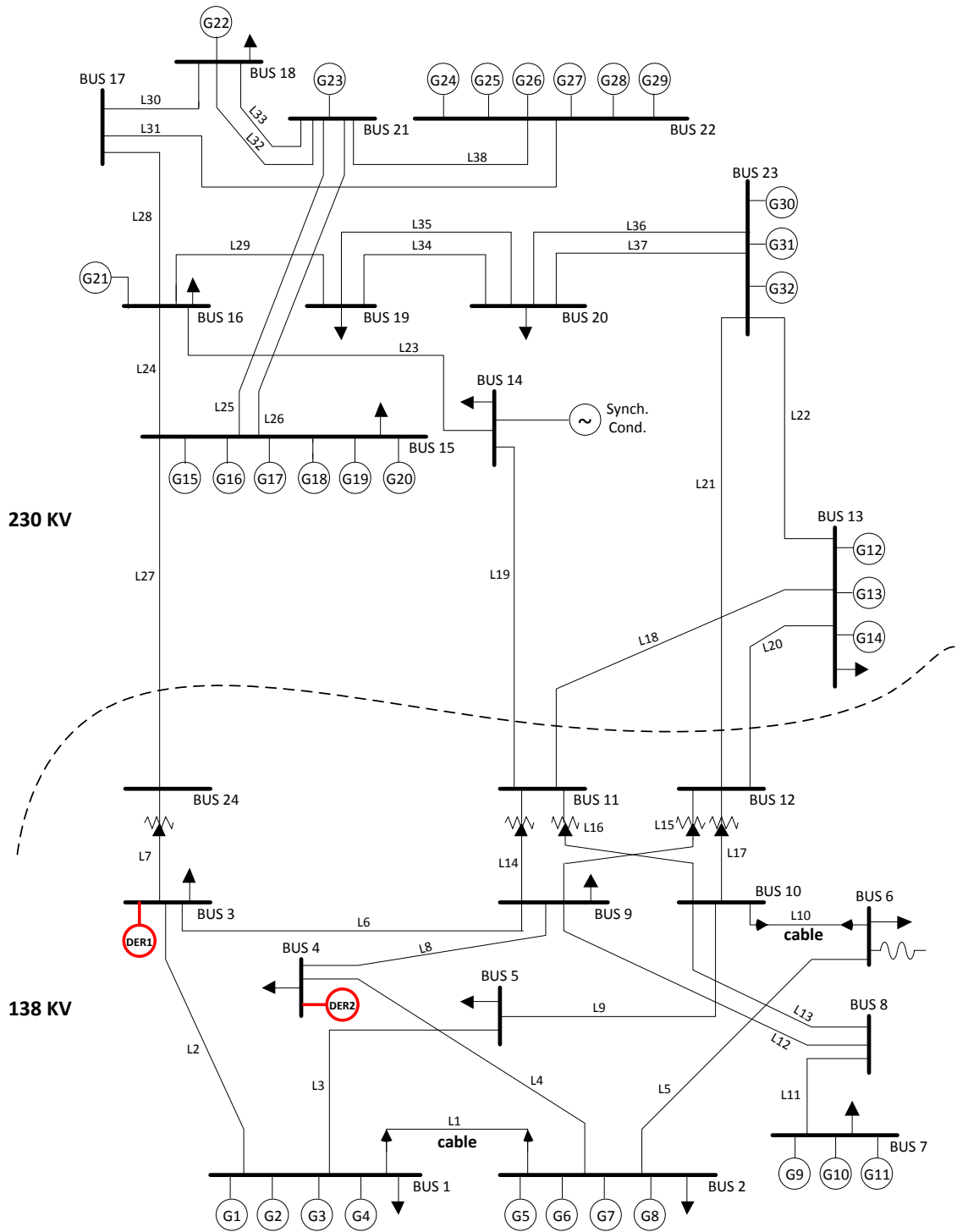


Figure 4.1: IEEE Reliability Test System with DERs

4.3.1 System Unreliability

Failure rates (λ) and the repair rates (μ) of each component of the IEEE Reliability Test System, as given in [28] are presented in Chapter 3, Tables 3.1 and 3.2. Further, λ and μ for the two DERs are presented in Table 4.1. The unavailability of each DER is calculated using (4.1).

The impact of the presence of DERs is observed in the formation of new combinations of minimal cut sets. Some of these new sets were previously lower order cut sets, and now they have changed to a higher order of cut sets. The loss of the older minimal cut set will no longer lead to an interruption. For example, a simultaneous outage of generator-23 and generator-30, or a maintenance operation on line-2 and line-7 will not cause a system failure anymore, because their cut sets change from second to third order, as shown in Table 4.3. These changes reduce the system unreliability (reduce LOLP), since, when the cut set order increases, its unavailability drops. The unreliability (LOLP) of the entire test system at a steady-state condition is now 0.1289 as shown in Table 4.3, i.e. the system reliability is improved from 84.38% to 87.11%. In the IEEE Reliability Test System under study, the bus-wise unreliabilities are also enhanced after adding DERs to the system. Table 4.4, Fig. 4.3 and Fig. 4.4 show the loss of load of all load buses and their new unreliability (LOLP) indices.

Table 4.1: Data and Steady-State Unavailability for DERs

Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	FOR
33	DER#1	1.00E-03	2.00E-02	4.76E-02
34	DER#2	1.00E-03	2.00E-02	4.76E-02

Table 4.2: Computation of Composite System Reliability with DERs using Minimal Cut Sets

Component	Order of Cut-Set	Set of Components in Minimal Cut Set	Minimal Cut Set Unavailability	Unavailability by Component and order of Minimal Cut Set
Generators Only	1st	None	0	0
	2nd	(12,22),(12,23),(12,32),(13,22),(13,23),(13,32),(14,22),(14,23),(14,32),(20,22),(20,23),(21,22),(21,23),(22,23),(22,30),(22,31),(22,32),(23,32)	(6.000E-3),(6.000E-3),(4.000E-3),(6.000E-3),(6.000E-3),(4.000E-3),(6.000E-3),(6.000E-3),(4.000E-3),(4.800E-3),(4.800E-3),(4.800E-3),(4.800E-3),(4.800E-3),(1.440E-2),(4.800E-3),(4.800E-3),(9.600E-3),(9.600E-3)	0.1104
	3rd	(1,23,30),(1,23,31),(2,23,30),(2,23,31),(3,4,22),(3,4,23),(3,7,22),(3,7,23),(3,8,22),(3,8,23),(3,9,22),(3,9,23),(3,10,22),(3,10,23),(3,11,22),(3,11,23),(3,20,32),(3,21,32),(3,23,30),(3,23,31),.....,(29,30,32),(29,31,32),(30,31,32)	(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-4),(4.800E-5),(4.800E-5),(4.800E-5),(4.800E-5),(4.800E-5),(9.600E-5),(9.600E-5),(9.600E-5),(9.600E-5),(9.600E-5),(9.600E-5),(6.400E-5),(6.400E-5),(9.600E-5),(9.600E-5),.....,(3.200E-5),(3.200E-5),(1.280E-4)	0.017389
Transmission Lines Only	1st	None	0	0
	2nd	(3,9),(4,8),(5,10),(11,13),(12,13),(19,23)	(1.461E-7),(1.828E-7),(7.211E-7),(1.719E-7),(2.520E-7),(2.335E-7)	EPS
	3rd	(1,4,10),(1,6,7),(1,6,27),(1,8,10),(1,10,11),(1,10,14),(1,10,15),(2,3,27),(2,5,7),(2,5,27),(2,6,7),(2,6,27),(2,7,9),(2,7,11),(2,7,13),(2,7,14),(2,7,15),(2,7,16),(2,7,17),(2,7,23),(2,7,27),.....,(25,26,28),(29,34,35),(29,36,37)	(2.57E-10),(2.86E-12),(9.78E-11),(2.37E-10),(8.69E-12),(8.69E-12),(1.13E-10),(4.80E-12),(1.64E-10),(3.80E-12),(1.30E-10),(3.40E-12),(3.00E-12),(4.40E-12),(1.32E-13),(1.32E-13),(1.32E-13),(1.32E-13),(4.18E-12),(1.16E-10),.....,(1.16E-10),(9.71E-11),(7.77E-11)	EPS
Generators & Transmission Lines	1st	N/A	0	0
	2nd	1 Gen + 1 Line (22,11),(23,11)	(4.108E-5),(4.108E-5)	EPS
	3rd	1 Gen + 2 Lines (3,2,7),(3,2,27),(4,2,7),(4,2,27),(5,1,10),(6,1,10),(7,1,10),(7,2,7),(7,2,27),(8,1,10),(8,2,7),(8,2,27),(12,2,7),(12,2,27),(13,2,7),(13,2,27),(14,2,7),(14,2,27),(20,2,7),(20,2,27),.....,(32,31,38),(33,2,7),(33,2,27)	(1.75E-10),(5.988E-9),(1.75E-10),(5.988E-9),(5.770E-8),(5.770E-8),(1.154E-8),(1.75E-10),(5.988E-9),(1.154E-8),(1.75E-10),(5.988E-9),(4.38E-10),(1.497E-8),(4.38E-10),(5.988E-9),(4.38E-10),(1.497E-8),(3.51E-10),(1.198E-8),.....,(3.061E-8),(4.18E-10),(1.43E-08)	EPS
	3rd	2 Gens + 1 Line (1,32,11),(2,32,11),(3,32,11),(4,32,11),(5,32,11),(6,32,11),(7,32,11),(8,32,11),(9,10,11),(9,10,13),(9,11,11),(9,11,13),(10,11,11),(10,11,13),(12,13,11),(12,14,11),(13,14,11),(20,32,11),(20,32,13),(21,32,11),.....,(31,32,11),(31,32,13),(32,33,11)	(2.739E-6),(2.739E-6),(5.478E-7),(5.478E-7),(2.739E-6),(2.739E-6),(5.478E-7),(5.478E-7),(5.478E-7),(8.032E-7),(5.478E-7),(8.032E-7),(8.032E-7),(8.559E-7),(8.559E-7),(1.096E-6),(1.096E-6),(1.096E-6),.....,(1.096E-6),(1.096E-6),(1.30E-06)	0.000112114
SYSTEM UNRELIABILITY (LOLP)				0.127985584

*EPS: Very small number

Table 4.3: Bus-Wise Unreliability and Loss of Load

Bus No.	Total Loss of Load (p.u.)	Unreliability (LOLP)	Bus No.	Total Loss of Load (p.u.)	Unreliability (LOLP)
3	29.6892	0.073413335	13	0.202	0.000125
4	8.926	0.053176151	14	22.154	0.05007643
5	1.01	0.000769242	15	0.726	0.000192
6	9.934	0.065342417	18	7.372	0.024205612
7	3.361	0.00161214	19	4.093	0.0096
8	30.469	0.016622607	20	1.899	0.000128012
9	1.277	3.1031E-08			

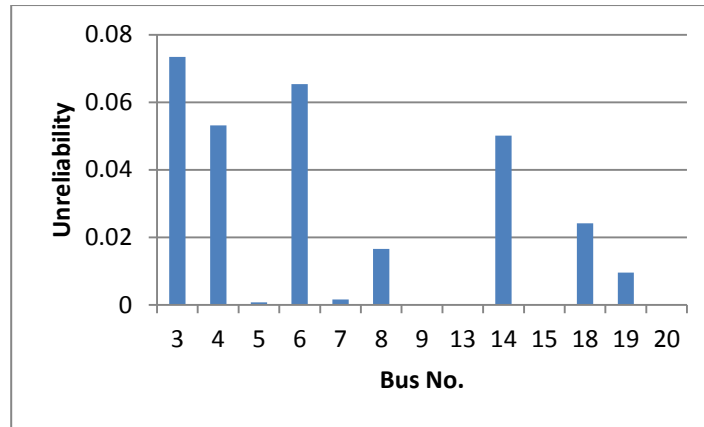


Figure 4.2: Bus-Wise Unreliability (LOLP) Indices

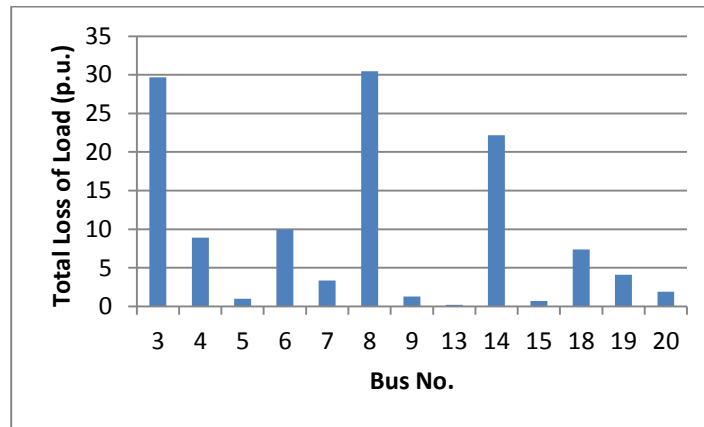


Figure 4.3: Bus-Wise Total Loss of Load

4.3.2 System Operational Risk

It is assumed that the DERs are independent of the standstill time and the full power of the DERs is available within one hour of activating a startup event. It is further assumed that the start-up failure probability is equal to 4%. The unavailabilities of the DERs are calculated using (4.2). For a lead time of 10 hours this results in an unavailability of 0.0482. The components unavailability as a function of the lead-time are presented in Chapter 3, Tables 3.6 and 3.7, and also shown in Figs. 3.7 and 3.8. The DERs unavailability as a function of the lead-time are calculated and presented in Table 4.4 and shown in Fig. 4.4.

The unavailability of the minimum cut sets is calculated, as before, with the results as shown in Table 4.5. In Fig. 4.5, it is observed that the unavailabilities of the minimal cut sets, that have a DER in their combination, have reduced because of the increase in their cut set order. The impact of the DERs on the operational risk is calculated to be 0.0015306. The operational risk as a function of time is shown in Fig. 4.6. The operational risk for a lead-time of 10 hours is reduced from about 0.175% to about 0.153% because of the penetration of the DERs. The load bus unreliabilities are also improved after penetrating the DERs to the system. Table 4.6, Fig. 4.7 and Fig. 4.8 show the loss of load of all load buses and their new unreliabilities.

Table 4.4: Generators and DERs Data and Unavailability after 10 Hours

Element Number	Description	Failure Rate (λ) 1/hr	Repair Rate (μ) 1/hr	FOR
33	DER#1	1.00E-03	2.00E-02	4.82E-02
34	DER#2	1.00E-03	2.00E-02	4.82E-02

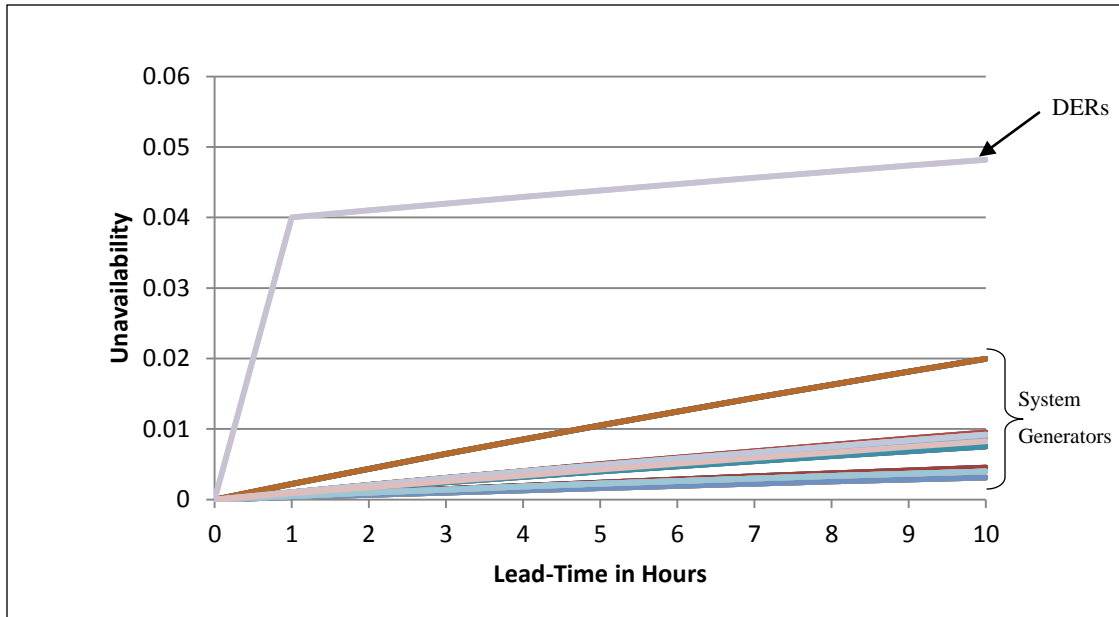


Figure 4.4: Generator Unavailability as a Function of the Lead-Time with DERs

Table 4.5: Computation of System Operational Risk with DERs using Minimal Cut Sets

Component	Order of Cut-Set	Set of Components in Minimal Cut Set	Minimal Cut Set Unavailability at $t = 10$ hr	Unavailability by Component and order of Minimal Cut Set
Generators Only	1st	None	0	0
	2nd	(12,22),(12,23),(12,32),(13,22),(13,23),(13,32),(14,22),(14,23),(14,32),(20,22),(20,23),(21,22),(21,23),(22,23),(22,30),(22,31),(22,32),(23,32)	(8.31E-05),(8.31E-05),(7.82E-05),(8.31E-05),(8.31E-05),(7.82E-05),(8.31E-05),(8.31E-05),(7.82E-05),(8.03E-05),(8.03E-05),(8.03E-05),(8.03E-05),(7.67E-05),(8.03E-05),(8.03E-05),(7.21E-05),(7.21E-05)	0.0014359
	3rd	(1,23,30),(1,23,31),(2,23,30),(2,23,31),(3,4,22),(3,4,23),(3,7,22),(3,7,23),(3,8,22),(3,8,23),(3,9,22),(3,9,23),(3,10,22),(3,10,23),(3,11,22),(3,11,23),(3,20,32),(3,21,32),(3,23,30),(3,23,31),.....,(29,30,32),(29,31,32),(30,31,32)	(1.6E-6),(1.6E-6),(1.6E-6),(1.6E-6),(1.775E-7),(1.775E-7),(1.775E-7),(1.775E-7),(1.775E-7),(1.775E-7),(2.966E-7),(2.966E-7),(2.966E-7),(2.966E-7),(2.966E-7),(3.403E-6),(3.403E-6),(3.616E-6),(3.616E-6),.....,(2.996E-7),(2.996E-7),(6.930E-7)	9.02951E-05
Transmission Lines Only	1st	None	0	0
	2nd	(3,9),(4,8),(5,10),(11,13),(12,13),(19,23)	(5.84E-08),(7.31E-08),(1.13E-07),(6.87E-08),(1.01E-07),(8.33E-08)	EPS
	3rd	(1,4,10),(1,6,7),(1,6,27),(1,8,10),(1,10,11),(1,10,14),(1,10,15),(2,3,27),(2,5,7),(2,5,27),(2,6,7),(2,6,27),(2,7,9),(2,7,11),(2,7,13),(2,7,14),(2,7,15),(2,7,16),(2,7,17),(2,7,23),(2,7,27),.....,(25,26,28),(29,34,35),(29,36,37)	(1.88E-11),(4.76E-13),(1.72E-11),(1.73E-11),(5.69E-13),(5.69E-13),(2.69E-11),(1.09E-12),(3.92E-11),(8.59E-13),(3.10E-11),(7.69E-13),(6.78E-13),(9.59E-13),(2.67E-14),(2.67E-14),(2.67E-14),(2.67E-14),(8.93E-13),(2.77E-11),.....,(2.48E-11),(2.07E-11),(1.66E-11)	EPS
Generators & Transmission Lines	1st	N/A	0	0
	2nd	<div><div>1st Gen + 1 Line</div><div>(22,11),(23,11)</div></div>	(1.90E-06),(1.90E-06)	EPS
	3rd	<div><div>1st Gen + 2 Lines</div><div>(3,2,7),(3,2,27),(4,2,7),(4,2,27),(5,1,10),(6,1,10),(7,1,10),(7,2,7),(7,2,27),(8,1,10),(8,2,7),(8,2,27),(12,2,7),(12,2,27),(13,2,7),(13,2,27),(14,2,7),(14,2,27),(20,2,7),(20,2,27),.....,(32,31,38),(33,2,7),(33,2,27)</div></div>	(1.41E-11),(5.09E-10),(1.41E-11),(5.09E-10),(1.33E-9),(1.33E-9),(3.01E-10),(1.41E-11),(5.09E-10),(3.01E-10),(1.41E-11),(5.09E-10),(2.98E-11),(1.073E-9),(2.98E-11),(1.073E-9),(2.98E-11),(1.073E-9),(2.98E-11),(1.073E-9),(2.87E-11),(1.037E-9),.....,(1.125E-9),(1.51E-10),(5.45E-9)	EPS
		<div><div>2nd Gens + 1 Line</div><div>(1,32,11),(2,32,11),(3,32,11),(4,32,11),(5,32,11),(6,32,11),(7,32,11),(8,32,11),(9,10,11),(9,10,13),(9,11,11),(9,11,13),(10,11,11),(10,11,13),(12,13,11),(12,14,11),(13,14,11),(20,32,11),(20,32,13),(21,32,11),...,(31,32,11),(31,32,13),(32,33,11)</div></div>	(3.554E-8),(3.554E-8),(8.031E-9),(8.031E-9),(3.554E-8),(3.554E-8),(8.031E-9),(8.031E-9),(1.225E-8),(1.796E-8),(1.225E-8),(1.796E-8),(1.225E-8),(1.796E-8),(1.95E-8),(1.95E-8),(1.95E-8),(1.636E-8),(2.399E-8),(1.636E-8),.....,(1.636E-8),(2.399E-8),(8.596E-8)	EPS
SYSTEM OPERATIONAL RISK				0.001531924

*EPS: Very small number

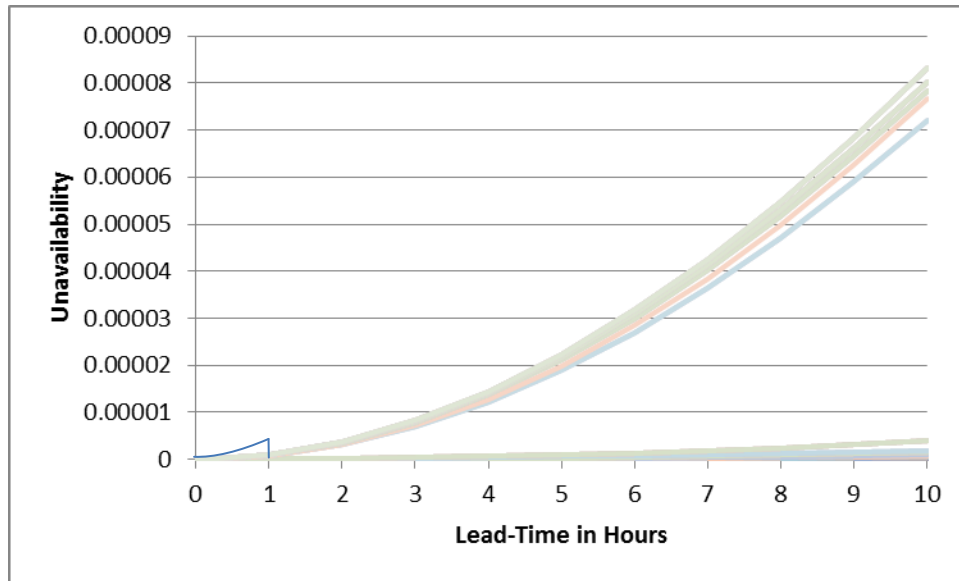


Figure 4.5: System MCS Unavailability as a Function of the Lead-Time with DERs

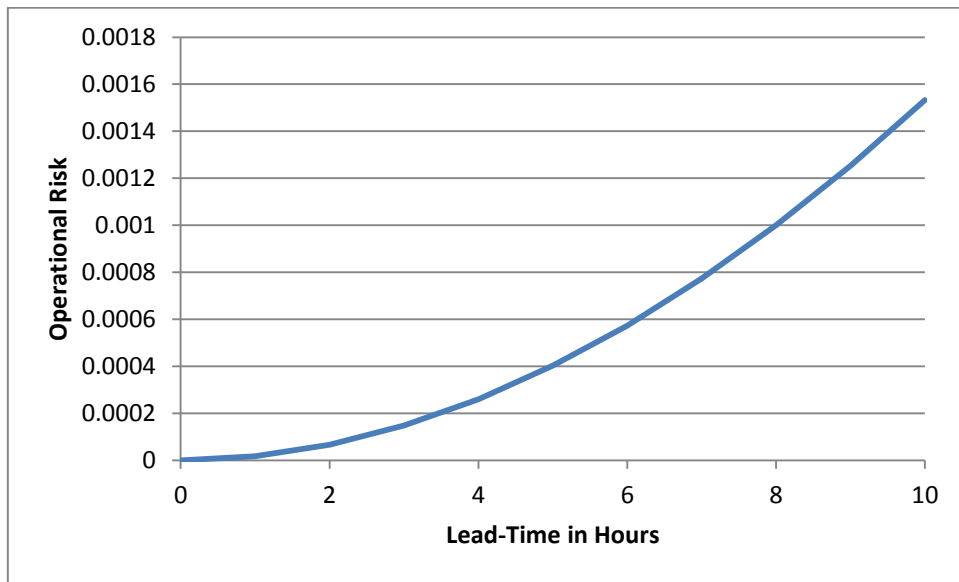


Figure 4.6: System Operational Risk as a Function of the Lead-Time with DERs Neglecting Changes in System Operational Conditions

Table 4.6: Bus-Wise Operational Risk and Total Loss of Load

Bus No.	Total Loss of Load (p.u.)	Operational Risk	Bus No.	Total Loss of Load (p.u.)	Operational Risk
3	29.6892	0.000934191	13	0.202	8.552E-07
4	8.926	0.000606401	14	22.154	0.000568869
5	1.01	4.15806E-06	15	0.726	7.363E-07
6	9.934	0.000846034	18	7.372	0.000149668
7	3.361	4.93959E-06	19	4.093	7.214E-05
8	30.469	0.000101344	20	1.899	6.94054E-07
9	1.277	1.1738E-09			

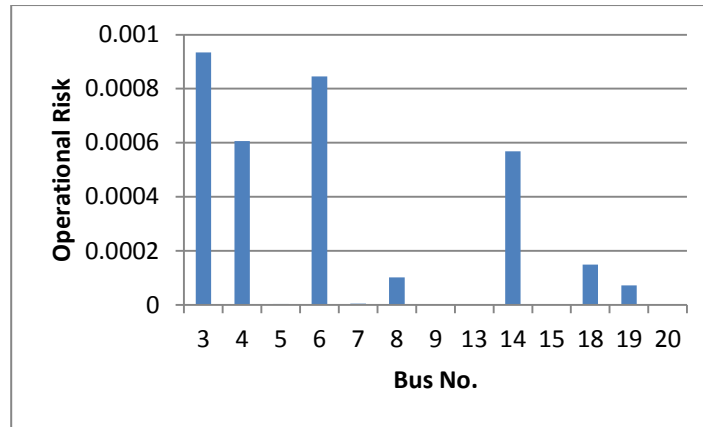


Figure 4.7: Bus-Wise Operational Risk

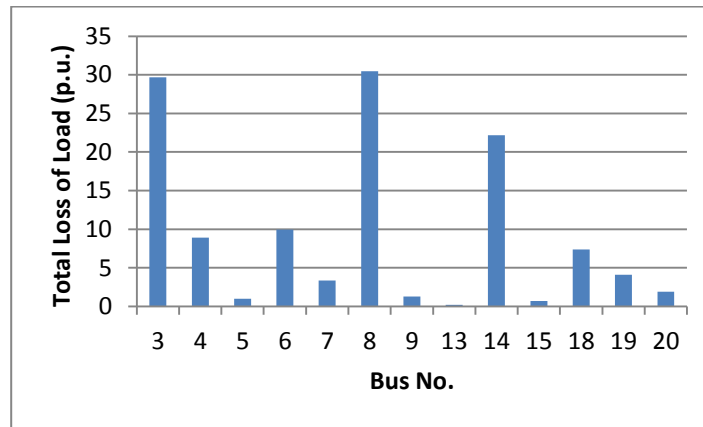


Figure 4.8: Bus-Wise Total Loss of Load

4.4 Summary

In this chapter, a reliability analysis of a composite power system with penetration of DERs, is presented. The mathematical model is described and it is applied to the IEEE Reliability Test System. Once the component failure probability is calculated using the data of the failure and repair rates, the minimal cut set evaluation is implemented. Network constraints are imposed to guarantee acceptable reliability level. Results showed the improvements and the positive impact of the DERs on power system reliability.

Chapter 5

Conclusions and Future Work

5.1 Summary of the Thesis

Reliability studies play an important role in ensuring the delivery of power to customers. Developing more efficient and intelligent power system reliability assessment techniques plays a key role in improving the system reliability. This thesis presents a reliability analysis framework for a composite power system to assess long-term reliability of the system and a real-time risk assessment method.

In Chapter 1, the motivation for this research is presented. This is followed by a brief review of the relevant literature addressing reliability assessment in power systems. Thereafter, the objectives of this research are presented.

In order to have a better understanding of the reliability concepts, a brief background on some basics definitions and reliability evaluation methods is described in Chapter 2. The concepts are illustrated by application to a small complex configuration describing the logical connections between components. Thereafter, a brief introduction of the types of component outages in a composite power system (HL-II) is presented. Finally, a quick overview on operation risk assessment of power systems is presented.

In Chapter 3, a comprehensive reliability and operational risk assessment method for composite power systems is proposed to help evaluate the system performance over several years (planning) and during a specific time (operation). An OPF based method is used to determine the minimal cut sets. Once the component failure probability is calculated using the data of failure and repair rates, the minimal cut set evaluation is implemented by using the approximate equations to evaluate system-wide and nodal reliability indices. The proposed method applied to the IEEE Reliability Test System. Network constraints are imposed to guarantee acceptable reliability levels.

Chapter 4 examines the impact of DERs on the reliability and the operational risk of the composite power system based on the same approach discussed in the previous chapter. The mathematical model is also applied to the IEEE Reliability Test System. Results demonstrate the improvement and the positive impact of the DERs on the power system reliability.

5.2 Main Contributions

The main contributions of the research presented in this thesis are as follows:

1. A comprehensive assessment framework for long-term reliability and real-time operational risk analysis of composite power systems is proposed. The framework determines parameters for reliability considering minimal cut sets of system components. The main challenge in implementing the method is determining the minimal cut sets. The proposed method consists of two steps:
 - Develop an AC-OPF to determine the minimal cut sets.
 - Use the obtained cut sets to compute the reliability and risk indices of the components that construct the minimal cut set using basic probability theory.
2. The proposed method moves a step further to understand the reliability of serving customers at a specific load bus. Each minimal cut set, causing loss of load at a certain load bus, needs to be stored. The nodal reliability and risk indices are then computed after that using the stored minimal cut sets.
3. In order to investigate the impact of the rapid penetration of alternative energy sources on reliability, DERs are penetrated to the system under study. Results show the positive effect of the DERs in increasing the system reliability and operational risk.

5.3 Scope for Future Work

The possible extensions to the present work are outlined below:

1. Apply the developed method to radial distribution systems by using system reduction techniques to speed up reliability calculations. There is a need to identify those areas which may be replaced by a simple equivalent, by reducing the number of nodes without significantly distorting the original system.
2. Use alternative techniques such as Monte Carlo simulation and heuristic methods to incorporate the outages in composite system reliability and compare the results.
3. Improve the Markov model to include dependent outages in reliability calculations, such as fluctuating weather and derated generation outages.
4. Improve the failure rate modeling of the system components by using different distribution functions, such as Weibull Distribution and Curve Fitting, instead of using the Exponential Distributions. Based on the Exponential Distribution, multipliers are used to reflect the rate change of the failures. On the other hand, the goal of the Curve Fitting function, for example, is to find the rate values that most closely match the data.

Appendix A

IEEE Reliability Test System Data

Table A. 1: Generating Unit Location and Capability [28]

Generator Number	Bus	P_G^{\max} MW	Q^{\max} MVar	Q^{\min} MVar
1	1	20	10	0
2	1	20	10	0
3	1	76	30	-25
4	1	76	30	-25
5	2	20	10	0
6	2	20	10	0
7	2	76	30	-25
8	2	76	30	-25
9	7	100	60	0
10	7	100	60	0
11	7	100	60	0
12	13	197	80	0
13	13	197	80	0
14	13	197	80	0
15	15	12	6	0
16	15	12	6	0
17	15	12	6	0
18	15	12	6	0
19	15	12	6	0
20	15	155	80	-50
21	16	155	80	-50
22	18	400	200	-50
23	21	400	200	-50
24	22	50	16	-10
25	22	50	16	-10
26	22	50	16	-10
27	22	50	16	-10
28	22	50	16	-10
29	22	50	16	-10
30	23	155	80	-50
31	23	155	80	-50
32	23	350	150	-25

Table A. 2: Voltage Correction Devices [28]

Device	Bus	MVar Capability
Synchronous Condenser	14	50 Reactive 200 Capacitive
Reactor	6	100 Reactive

Table A. 3: Generator Reliability Data [28]

Unit Size (MW)	Unit Type	MTTF	MTTR	Forced Outage Rate
		(Hour)	(Hour)	
12	Oil/Steam	2940	60	0.02
20	Oil/CT	450	50	0.1
50	Hydro	1980	20	0.01
76	Coal/Steam	1960	40	0.02
100	Oil/Steam	1200	50	0.04
155	Coal/Steam	960	40	0.04
197	Oil/Steam	950	50	0.05
350	Coal/Steam	1150	100	0.08
400	Nuclear	1100	150	0.12

MTTF = mean time to failure = λ^{-1}

MTTR = mean time to repair = μ^{-1}

Forced Outage Rate = $MTTR / (MTTF + MTTR)$

Table A. 4: Bus Load Data [28]

Bus Number	Load	
	MW	MVAr
1	108	22
2	97	20
3	180	37
4	74	15
5	71	14
6	136	28
7	125	25
8	171	35
9	175	36
10	195	40
13	265	54
14	194	39
15	317	64
16	100	20
18	333	68
19	181	37
20	128	26
Total	2850	580

Table A. 5: Transmission Line Length, Reliability, Impedance, and Rating Data [28]

Line Number	From Bus	To Bus	Length (mile)	λ (1/yr)	MTTR (hours)	Impedance P.U. /100 MVA Base			Normal Rating (MVA)
						R	X	B	
1	1	2	3	0.24	16	0.003	0.014	0.461	175
2	1	3	55	0.51	10	0.055	0.211	0.057	175
3	1	5	22	0.33	10	0.022	0.085	0.023	175
4	2	4	33	0.39	10	0.033	0.127	0.034	175
5	2	6	50	0.48	10	0.05	0.192	0.052	175
6	3	9	31	0.38	10	0.031	0.119	0.032	175
7	3	24	0	0.02	768	0.002	0.084	0	400
8	4	9	27	0.36	10	0.027	0.104	0.028	175
9	5	10	23	0.34	10	0.023	0.088	0.024	175
10	6	10	16	0.33	35	0.014	0.061	2.459	175
11	7	8	16	0.3	10	0.016	0.061	0.017	175
12	8	9	43	0.44	10	0.042	0.161	0.044	175
13	8	10	43	0.44	10	0.043	0.165	0.045	175
14	9	11	0	0.02	768	0.043	0.165	0.045	175
15	9	12	0	0.02	768	0.002	0.084	0	400
16	10	11	0	0.02	768	0.002	0.084	0	400
17	10	12	0	0.02	768	0.002	0.084	0	400
18	11	13	33	0.4	11	0.006	0.048	0.1	500
19	11	14	29	0.39	11	0.005	0.042	0.088	500
20	12	13	33	0.4	11	0.006	0.048	0.1	500
21	12	23	67	0.52	11	0.012	0.097	0.203	500
22	13	23	60	0.49	11	0.011	0.087	0.182	500
23	14	16	27	0.38	11	0.005	0.059	0.082	500
24	15	16	12	0.33	11	0.002	0.017	0.036	500
25	15	21	34	0.41	11	0.006	0.049	0.103	500
26	15	21	34	0.41	11	0.006	0.049	0.103	500
27	15	24	36	0.41	11	0.007	0.052	0.109	500
28	16	17	18	0.35	11	0.003	0.026	0.055	500
29	16	19	16	0.34	11	0.003	0.023	0.049	500
30	17	18	10	0.32	11	0.002	0.014	0.03	500
31	17	22	73	0.54	11	0.014	0.105	0.221	500
32	18	21	18	0.35	11	0.003	0.026	0.055	500
33	18	21	18	0.35	11	0.003	0.026	0.055	500
34	19	20	27.5	0.38	11	0.005	0.04	0.083	500
35	19	20	27.5	0.38	11	0.005	0.04	0.083	500
36	20	23	15	0.34	11	0.003	0.022	0.046	500
37	20	23	15	0.34	11	0.003	0.022	0.046	500
38	21	22	47	0.45	11	0.009	0.068	0.142	500

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